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MONTANA.

Several shipments of interesting minerals from Montana have lately been received, a number of the most notable being reviewed below. The great mineral wealth of this district is well-known, and to its present activity is due the increasing number of mineral species reported.

CRYSTALLIZED RHODOCHROSITE. Characteristic rhombohedrons of a rich pink in aggregates, clean cut and perfect. They lack the translucency of the Colorado find, but in other features closely resemble the material from that state. Crystals vary in size from ¼ to an inch and more across with corresponding thickness. Occasionally, groups are coated with a white drusy quartz making an association both interesting and attractive. Prices 50c. to $2.00. Massive 50c. per lb.

CRYSTALLIZED ENARGITE. A few neat groups of the rare copper sulpharsenate, illustrating habit and modifications. Crystallized metallic minerals find quick sale, and it is confidently expected that at our low scale of prices, the best of the lot will be immediately sold. Prices 50c. to $3.00. Massive 75c. per lb.

AMETHYST. Our correspondent spent several months at the Amethyst locality, and after much difficulty succeeded in securing what are considered the finest specimens as yet found. The types show parallel growth, interesting capping forms and single doubly terminated crystals. Color is good, generally deepening at the tips. Convenient cabinet sizes—50c. to $4.00.

TOURMALINITIC QUARTZ.

Two shipments of this odd occurrence permit us to describe its varied habits and strange associations. In appearance the crystals are black, due to the fine network of included Tourmaline hairs, which occasionally are widely separated, reminding one of the more common "flêches d'amour." A peculiar "stem" extends beyond the base of some crystals, around which the Quartz seems to have formed. The Tourmaline is often isolated on the "stem," appearing in bristling tufts and masses. The very fact that these specimens differ widely from the usual Quartz forms, makes them attractive to the collector, but aside from this point they combine such interesting features scientifically, that a steady demand is anticipated. Prices 50c. to $3.50 in 2 and 4 inch sizes.

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THE

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[FOURTH SERIES.]

ART. IX.—*Experiments on High Electrical Resistance*, Part II; by OGDEN N. ROOD, Professor of Physics in Columbia University.

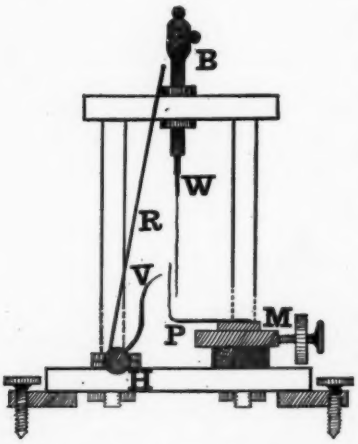
New method of measuring high electrical resistance.—In the first part of this paper.* I have given a general account of a mode of measuring electrical resistances which are far too large to be dealt with by ordinary methods. Since then the electrometer, there indicated, has been improved, and a large number of units of resistance has been constructed. Experiments have also been made with regard to the best modes of using them, and, in particular, with respect to the best way of building up a set of high resistances, from a low, but known, resistance. Attention has been paid to electrical leakage, to the composition of the units, to their reliability during short and long intervals of time. Their behavior under different electromotive forces has been somewhat attended to, and is still under examination. A large number of details have presented themselves, and it will require considerable time before they all can be satisfactorily studied. Meanwhile this much has been ascertained: it is possible to take a low standard of resistance, measured by old methods, employing fifty or a hundred volts, and from this to build up a set of resistances, the highest of which shall be equal to fifteen or twenty millions of megohms, the original electromotive force being always adhered to. With such units of resistance it is of course easy to measure any resistance which lies within their range, employing always one and the same electromotive force. The reliability of such measurements can be judged from the results given at the end of this article. It is also certain that the

* This Journal, x, 285, 1900.

cores of the units about to be described are inaccessible to moisture, and that in ordinary weather there is no surface conduction that interferes with their accuracy. Less is known about the invariability of the units during long periods of time; this matter is being investigated by Mr. H. C. Parker, and the results thus far obtained are promising. If it should turn out that none of the varieties I have devised has this desirable quality, it would merely necessitate on each occasion where accuracy was required, a galvanometer measurement, and a building up process from it, involving two or three hours of extra labor, as changes in the units do not occur which would interfere with the ease of a building up process once established in all its details. In every case thus far examined, the resistance of the unit immediately after making it has increased with some regularity, week by week, but at a diminishing rate, and now at the present time, it is not my custom to employ units till they are at least three months old.

Electrometer.—The construction of this apparatus is shown in the sketch, fig. 1, where all the shaded portions are metallic, and the unshaded parts made of the best quality of ebonite. B is a brass binding post, to which may be attached the wire conveying the charge, also a Leyden jar or a small mica condenser. It terminates below in an aluminum leaf, bluntly pointed, and provided with a long hinge of gold leaf for flexibility. The latter is attached to a brass wire W, that slides into a well-fitting tube, from which it can readily be removed for repairs. P is a springy brass plate faced with platinum. When the apparatus has received a sufficient charge the aluminum leaf is attracted to P, and usually remains attached; V is a narrow strip of thin sheet brass fastened to an axis, not shown in the diagram, which is terminated by the milled head H; it can be made to slightly scrape the edge of the plate P, and communicate to it a trifling vibratory movement, which instantly detaches the gold-aluminum leaf. Mere contact from behind, even when made with some violence, is not sufficient for this pur-

1



pose. R is a stiff brass wire attached to the same axis with V, and serves to discharge the apparatus; it has an arm bent at a right angle which can be brought into contact with the binding post B. A single motion discharges the electrometer and sets the gold leaf free. The axis carrying R and V is connected with the ground. The micrometer, M, is provided with a millimeter screw and carries the plate P. In all of my recent experiments it was connected with the ground. The height of the electrometer from the table, to the top of the binding post, is 27^{cm}. It is covered by a glass case to prevent air currents, the case being in contact only with the ebonite base. When in actual use the apparatus is covered by a pasteboard box coated with tin-foil, through the sides of which suitable openings are made for observation, and for the introduction of the two wires that are attached to the binding post. The electrometer as thus described, with a striking distance of a half or of one millimeter, is suitable only for measuring resistances of a million megohms and upwards; lower resistances bring about a stroke in a very few seconds, and to measure these, its capacity must be increased.

Condensers.—A series of these have been employed, the greatest being a large Leyden jar with a capacity of ·00507 of a microfarad; four smaller jars with diminishing capacities and three small mica-condensers have been used. When the electrometer is once made, and a few suitable resistances, such a set of condensers can easily be produced. The mica-condensers were supported by silk threads that had been heated up in a bath of wax and rosin. A long wooden rod above the apparatus carried the mica-condensers and the units, when more than three or four of them were used in a series, and in this way excellent insulation was readily obtained.

Table of Capacities of Condensers.

	Capacity in microfarads.
Leyden jar, No. 1	·0050
" " 2	·0025
" " 3	·0013
" " 4	·0008
" " 5	·00044
Mica cond., No. 1	·00024
" " 2	·00014
" " 3	·000037
Electrometer	·000014

It may be remarked that the apparatus described in this paper can easily be used to determine the capacities of quite small condensers, and of conductors or semiconductors sus-

pended in the air by insulating threads. With a set of condensers like the above, the capacity of any condenser within their range can be quickly obtained.

Units of resistance.—After a number of experiments in different directions, it was decided to employ peroxide of manganese painted on strips of blue cobalt glass, which was found to be a much better insulator than colorless plate glass. The mode of construction, briefly, is as follows: washed commercial peroxide of manganese is painted with a brush on cobalt glass, the edges of the strip being avoided, and it is then dried with gentle heat. The layer thus obtained is quite adherent in spite of the fact that only water is used in making the mixture, and it can be laid on with different thicknesses and have desirable breadths. The two ends of the glass slip are then wrapt with tin-foil, and about 7^{cm} of the foil allowed to remain free at the two ends to hold the unit when it is immersed in a rosin-wax bath. Fine brass wire is used to prevent the unit from unrolling itself when it is lifted up. The rosin-wax bath contains only so much wax as does not dim the polish of the finished unit, the polish of the bath itself, when it cools, being about half destroyed. Pure yellow wax was employed, and the temperature of the bath was about 150° C. Immersion in this bath drives out air and moisture, and as more than half of the tin-foil is also immersed, it results that the unit is embedded in the insulating substance, conduction to its interior being furnished by the free tin-foil at the ends. Afterwards, while still as hot as allowed handling, the units were finished with a hot knife, care being taken repeatedly to press down the foil at the places of contact. The superfluous rosin being removed, the remainder of the tin-foil was wrapt around the ends of the units and suitable terminals of sheet brass supplied. These again were wound with flexible iron wire having a diameter of $0{\cdot}4^{mm}$. In this operation as much force as was considered safe was employed, in order to secure fixity, and to make sure that the pressure exerted on the foil should be far greater than any experienced afterwards in the use of holders to connect the units, or to introduce them into the circuit. It will be seen that the units are completely inaccessible to moisture, and experiments have shown that in ordinary weather their surface conduction, practically, is zero. Three-quarters of the insulating surface between the terminals have been wrapt with tin-foil without lowering the resistance, even when the hygrometer indicated 60 per cent of moisture.

The process which has been explained is suitable for the production of units having resistances from one up to ten thousand megohms. For higher units the oxide of manganese

is to be diluted with Chinese vermilion, when good substantial layers can still be used. If the amount of the oxide of manganese be reduced as low as twenty per cent, as high resistances can be obtained as any that have been handled by me. The thickness of oxide of manganese giving a resistance of only one megohm was not measured, but must have been less than one millimeter. Some of these low units, of from one to a hundred megohms resistance, were given to Mr. H. C. Parker, who is engaged in testing their constancy; he finds that they exhibit no polarization, neither have I, experimenting with higher units, detected anything of this kind, by which is meant that in neither case does the resistance increase while the current is flowing.

As already indicated, the best results were obtained by the use of an insulating material of rosin with a small percentage of yellow wax. Paraffin was also tried, but even with low units exhibited a tendency to break down and lower its resistance, temporarily; plain yellow wax was better, but both of them were liable to break down rather suddenly when the resistances were quite high. Plain peroxide of manganese on cobalt glass, not insulated by immersion in any bath, answered admirably in dry weather for rather low units (under 50,000 Ω) but in damp weather became quite unreliable.

The units employed by me were 10^{cm} long, 2^{cm} broad, each brass terminal having a length of 2^{cm}. For the higher resistances they might be made two or three times as long with advantage.

Battery.—In the earlier experiments the street current was employed with an E.M.F. of 112 volts, but was found to be unsatisfactory owing to its small variations; an arrangement of 200 cells or tubes with copper and zinc and very dilute acid is preferable, but the so-called dry cells are far better, as with such high resistances their electromotive force is almost perfectly constant, and they need no care.

Leakage.—It would appear tolerably certain that when the electrometer, with or without a condenser, is first employed, a small amount of electricity is, so to speak, absorbed by it, so that with a given unit, the time of a stroke becomes in a few minutes somewhat shorter and then remains constant. This leakage reduces itself to a minimum, but there is reason to believe that some always remains. For example, if two equal units are joined in a series, and matters are arranged so that the time of the stroke does not exceed thirty or forty seconds, then this joint time will be equal to the sum of their times used separately. If, however, this joint time is allowed to rise as high as one or two minutes, then it always exceeds the sum of the separate times by anything from four to ten per cent or

more, according to the dryness of the atmosphere. Various expedients were tried to measure and allow for this loss, but it was finally decided to compare together only resistances that were nearly equal, which is not a matter of difficulty after one has accumulated a stock of resistances, those which had been accounted failures now becoming useful. It is hardly necessary to add that if a unit, of about a given resistance, is desired, it is best, after due consideration of the case, to make five or six.

Manipulation.—One pole of the battery is connected with a contact maker which must be carefully insulated; from thence the current flows to the unit or resistance, and finally to the binding post, B. The electrometer having been discharged and the gold leaf set free, the wire R is allowed to remain for ten seconds in contact with post B, the current meanwhile flowing through the unit or resistance into the ground. As R and V are turned to the left the time is noted with a stop watch, and the leaf watched with a lens for the stroke, after which contact is immediately broken. An attempt is always made to arrange matters so that the time noted shall not be less than fifty nor more than eighty seconds, and in the case of two resistances which are being compared together, care is taken to make their times as nearly equal as possible, so as to avoid errors due to leakage. This is accomplished by the use of small subsidiary units of known resistance, which are added to either of the resistances under consideration as occasion may require. When a string of units is being compared with another string, the actual number of pieces in both cases should be the same, any deficiency being made up by the use of dummies which in all respects imitate real units, except that they have no resistance at all.

The second pole of the battery and the plate P are permanently connected with ground. The striking distance usually employed has been one millimeter throughout whole sets of experiments. Striking distances as large as seven millimeters can be used for regulating the time of the stroke, but it is better to manage this with the aid of condensers. In any case, the striking distance and the electromotive force of the battery should remain constant in a set of experiments, so that comparable results can be directly obtained without the use of coefficients. Not only can the units be used "in series," but also "in parallel," to measure smaller units.

Building up a set of resistances.—Mr. H. C. Parker measured for me a resistance which turned out to be 32,000 megohms, the electromotive force being 50 volts, and this was used as a base in building up a set which reached as high as fourteen million megohms. The plan for building up

having been arranged so that only units or sets of units of about the same resistance were compared together, it was possible to run through the entire series in three hours, and on April 20th this was done, and repeated on April 21st. The number of readings in no case exceeded three; the striking distance was one millimeter. The degree of accuracy obtainable under such circumstances can be judged from the table here given, where the resistances are expressed in megohms.

April 21st.	April 22d.
34,000	34,000
12,580	12,260
23,910	24,140
26,970	27,200
73,580	73,460
190,560	189,700
217,800	220,800
440,200	431,400
520,400	517,600
1,420,200	1,420,000
1,849,000	1,761,200
1,862,400	1,645,400
2,570,000	2,360,000
4,768,000	4,944,000
6,364,000	6,110,000
7,432,000	7,092,000
14,658,000	13,724,000

Columbia University, New York, June 1st, 1901.

Art. X.—*Mineralogical Notes;* by A. J. Moses.

(1) *Mercuric Iodide from New South Wales.*

Mr. Roy Hopping submitted to me for examination a small specimen of impure limonite from the Broken Hill Mines, which was coated upon one side with a thin layer of bright scarlet or minimum red color. Under the microscope this layer proved to be made up of little sharp-edged cubes about $1/10^{mm}$ on an edge and with slightly etched or roughened faces.

The only measurements possible were with the cross hairs. Any face brought squarely beneath the objective was either square or rectangular, or in a few observed instances not less than three and probably all four of the corners were replaced by lines equally inclined to the sides, that is corresponding to planes of the octahedron. Other faces showed linear markings parallel to both diagonals. Geometrically everything suggested the hexoctahedral class of the isometric system. No confirmation could be obtained with polarized light because of surface irregularities, while in an attempt to mount for grinding smooth the crystals became yellow and disappeared.

The following tests were made as to composition: A group of three or four crystals on a small mass of gangue was heated on a glass slide and watched, during the heating, through a hand glass. The crystals slowly and apparently completely volatilized without any visible change of color, and in a similar test in a narrow closed tube there was obtained a sublimate in part yellow but principally of the original bright red color, composed however of crystals which were rather needle-like than cubic. Two or three crystals placed on white paper and touched with a hot bead gave a sublimate corresponding exactly to the red and yellow sublimate of the iodides of mercury which is obtained by heating a mercury compound on a plaster tablet with bismuth flux.

To further check the presence of mercury, Mr. J. S. McCord made for me Behren's microchemical test for mercury, by first subliming two or three particles on a glass slide, dissolving in aqua regia, evaporating to dryness, and then adding to the dry substance a minute drop of each of ammonic sulphocyanide and cobalt nitrate, obtaining immediately the fine groups of blue diverging needles of $Co(CyS_2)$, $Hg(CyS_2)$. A parallel test was made with artificial mercuric iodide, and throughout the behavior was the same. The deep green flame of copper iodide was obtained when a salt of phosphorus bead saturated with copper oxide was touched to a crystal of the substance and again heated.

The examination suggests very definitely mercuric iodide in isometric crystals of the hexoctahedral (holohedral) class. The tetrahedral copper iodide, marshite, and the hexagonal silver iodide iodyrite* both occur at the Broken Hill Mines, and Mr. George Smith mentions† "iodyrite associated with a bright red mineral readily tarnishing on exposure and found to be sulphide of mercury." The specimen examined has been in my possession over a year and has not noticeably tarnished, but as the Castillos mineral coccinite is said to change on exposure, it may be that the supposed sulphide of mercury will prove to be the mercuric iodide here described.

The two descriptions of "coccinite," the Mexican iodide of mercury, differ notably from this material. That of Castillos† from Zimapan and Culebras is "in acute acicular rhombic pyramids 2–6^{mm} long, fine red to yellow, changing to greenish gray and dark green on exposure; transparent to translucent. In closed tube a sublimate of Hg_2Cl_2, white when cold and leaving residuum dull red hot, orange yellow cold and before blowpipe turns aurora red and disappears with odor like selenium."

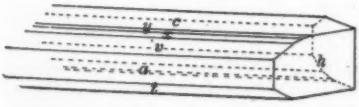
The older description by Del Rio in 1828 of material from Casas Viejas, Mexico, describes particles of reddish brown color, deeper red than cinnabar, on selenide of mercury, and adamantine in luster. Castillos says the specimens labelled by Del Rio contained no iodine.

It does not seem that the name coccinite can be at once applicable to the acute rhombic pyramids of Castillos and the cubes from Broken Hill Mines, and it is hoped that more material may be found for further examination.

(2) *New Forms on Bergen Hill Pectolite.*

Mr. F. V. Cruser, student in chemistry at Columbia University, found crystallized pectolite in an abandoned quarry at Weehawken, N. J., not far from the 42d Street Ferry. The crystals are needles about 3^{mm} long by only 3/10^{mm} broad, but fair reflections were obtained from two of the crystals by use of the Goldschmidt two-circle goniometer. The crystals were mounted with the axis $\bar{b}$ normal to the vertical circle and the results later transposed to conform to the conventional position with the axis $\dot{c}$ normal to the vertical circle.

1



As in previously recorded measurements the faces *a* (100)

* Proceedings Royal Society New South Wales, xxvi, 326 and 371, 1892; xxvii, 366, 1893: xxix, 49 and 189, 1895.

† Ibid., xxvii, 372.

‡ Dana's System, 6th Ed., 161.

yielded double signals. In the second crystal both faces of *a* yielded a bright and a dull signal, while in the first crystal the two signals were of equal intensity. The results following are derived from the bright signals of the second crystal.

Crystal 1 showed definitely the forms *a* (100), *c* (001), *v* (101) and *h* (540), while crystal 2 included in addition the forms *t* ($\bar{1}01$) and also two undescribed domes *x* (102) and the very flat dome *y*=(1·0·25), but does not include *v*. Figure 1 represents all the faces. The coördinate angles as obtained, after transposition to the conventional position, and the calculated coördinate angles are as follows:

	Measured.		Calculated.	
Face.	ϕ	ρ	ϕ	ρ
c (001)	90°	5° 23′	90°	5°20′
a (100)	90	90	90	90
v (101)	90	44 46	90	44 30
x (102)	90	30 27	90	30 30
y (1·0·25)	90	7 29	90	7 32½
t ($\bar{1}01$)	90	38 52	90	38 31
h (540)	48 36′	90 .	48 25	90

x, *y*, *t* and *h* were very minute faces yielding only faint images.

These values correspond to the angles

$ac=\beta=84°37'$. $av=45°14'$. $ax=59°33'$. $ay=82°31'$.
$a't=51°8'$. $ct=44°15'$. $hh'=82°48'$.

It is to be noted that a change of 3′ in the determination of the position of *a* would almost exactly reduce the values for *c*, *x* and *y* to the calculated values and would lessen the differences for *v* and *t*. This amount of divergence was easily possible with the double image. On the other hand, it is to be noted that the angle *ac* for the isomorphous wollastonite is 84° 30′, and that for pectolite it has been variously determined as 84° 35′,* 84° 30′,† 84° 40′‡ and may be said to be still uncertain.

(3) *New Forms on Atacamite Crystals from Chili.*

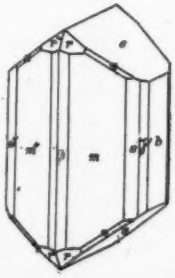
Mr. John C. F. Randolph, mining engineer, brought from the Brilladora Mine, Paposo, near the coast in the northern part of the Province of Atacama, several masses of crystals of of atacamite which differ sufficiently from previously examined crystals to warrant description.

* Greg and Lettsom, Mineralogy 1858, p. 213.
† DesCloizeaux, Mineralogie 1862, i, p. 547.
‡ Dana, System Mineralogy 1892, p. 373.

I have been unable to identify the material with any of the occurrences described by Darapsky,* and it is evidently not the material described from 'Sierra Gorda,' of which G. F. H. Smith states† "that crystals of atacamite are rarely doubly terminated and it is not therefore possible to determine by simple measurement whether the mineral is truly holohedral," whereas the Brilladora material consists almost entirely of doubly terminated crystals. The material examined by Brögger‡ shows greater similarity than any other both in the predominating faces of the crystals and in the porous ferruginous gangue and associated minute crystals of gypsum. Brögger, however, describes the crystals as in general "poorly formed with usually deeply striated vertical zone" and rounded edges and poor terminal planes, while the specimens from Brilladora Mine consist of well-formed crystals, the prism faces, though somewhat curved and irregular, are not striated and with few exceptions the crystals are doubly terminated.

Figure 2 shows the usual form of crystal except that the modifying prism planes are represented larger than in the actual crystals, in which they are little more than knife-edge width. The crystals are not tabular but are usually of approximately equal dimensions, parallel $\breve{a}$ and $\overline{b}$ and a little longer parallel $\breve{c}$. They range in size from $4\times4\times6^{mm}$ down. Under magnification the faces of the prism m (110) are seen to be

2



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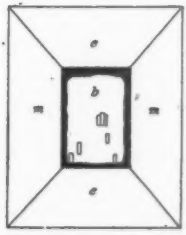


slightly curved and they yield only fair reflections. The face b (010), parallel to the cleavage, is in some crystals rough and bright, in other crystals smooth but dull and in others is centrally smooth and dull but with a narrow periphery (fig. 3) of bright material, as if the adjoining faces of m and e were covered with a thin veneer. This face b sometimes shows rectangular pittings similar to etch figures as also shown in fig. 3.

* Neues Jahrb. f. Min., ii, 1–18, 1889.
† Min. Mag., xii, 15, 1898.
‡ Zeitschr. f. Kryst, iii, 488, 1879.

On account of the minuteness of the modifying faces the measurements were made upon a Goldschmidt two-circle goniometer, and it will be noticed that the angles obtained deviate considerably from the calculated angles. The readings, however, agreed well with each other and the variation from the calculated angles are due principally to the fact that in orienting the crystals, in the absence of the basal plane, the prismatic zone was employed and in all recorded measurements either of Chilian crystals* or of those from Wallaroo, South Australia,† there have been marked variations in this zone, so great, even with the most perfect crystals, that the calculated angles only result as an average of many different crystals and measurements.

It may be noted that many of the crystals are interpenetrated and with some appearance of regularity. A careful measurement failed to prove any true twinning.

In the tabulation which follows I have calculated ϕ and ρ to the elements obtained by Smith,‡ viz: $\breve{a} : \bar{b} : \dot{c} = 0{\cdot}66130 : 1 : 0{\cdot}75293$, and have used as a meridian a position midway between the reading for 110 and $\bar{1}10$, assuming this to be the true position of 010. Two new prisms were found, (150) designated by γ, which occurred with s (120) on one crystal in easily visible faces, the other (210) designated by β and of which two very narrow faces were observed which yielded distorted reflections. All of the measurements with the exception of γ (150) were obtained from one bright little crystal, almost octahedral in shape and approximately $1\frac{1}{2}^{mm}$ parallel to $\breve{a}$, $\bar{b}$ or $\dot{c}$. Two other larger crystals were measured and showed the same forms, but being less bright gave poorer reflections. On one of these the (150) prism was found.

The observed and calculated coördinates were:

	Observed.		Calculated.	
Face.	ϕ	ρ	ϕ	ρ
b 010	0° 08′	90°	0°	90°
γ 150	16 35	90	16 50′	90
s 120	37 12	90	37 05	90
m 110	56 26	90	56 31	90
β 210	72 05	90	71 42	90
e 011	0 02	37 4′	0	36 59′
n 121	37 04	61 58	37 05	62 06
r 111	56 21	53 39	56 31	53 46

* See previous references

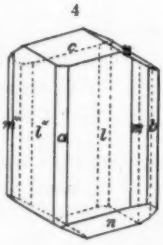
† Zepharovich, Ber. Ak. Wien, lxiii, 6, 1871; Klein, Neues Jahrb. f. Min., 1869, 347; E. S. Dana, Min. Mitth., ii, 103, 1874.

‡ L. c.

(4) *Realgar Crystals from Snohomish County, Washington.*

The described occurrences of realgar in this country have, I believe, none of them yielded measurable crystals.* Certainly this is the case with the stalagmitic material from Yellowstone Park described by Weed and Pirsson† and the bladed, confusedly aggregated crystals found by Blake† beneath the lava in Iron County, Utah.

Through the kindness of Prof. J. F. Kemp I received a specimen of crystallized realgar from a vein two to four inches thick in a tunnel of the Penn Mining Co., Monte Cristo Mining District, Snohomish County, Washington. The crystals rest upon a thin layer of nearly black tarnished marcasite.



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Most of the crystals are long, prismatic, and so embedded that little more can be seen than the deeply striated faces of the prism zone, sometimes 30^{mm} in length. Some smaller crystals, however, show also three terminal planes suggesting a flat rhombohedron.

The best of these crystals, approximately 2×2^{mm} cross section, was carefully detached and measured with the two-circle goniometer. The occurring forms were, as shown in fig. 4:

Pinacoids.—*a* (100) and *b* (010) small but yielding fair signals. *c* (001) large but dull.

Prisms.—The prism faces were striated, but two faces of *m* (110) yielded faint separate signals, three faces of *l* (210) yielded moderately good signals, and there were indications of faces of *h* (670).

Pyramids.—The faces of *n* ($\bar{2}12$) yielded good reflections.

The coördinate angles measured and calculated were:

	Measured.		Calculated.	
Face.	ϕ	ρ	ϕ	ρ
c (001)	89° 32′	23° 57′	90°	23°55′
b (010)	0	90	0	90
a (100)	90 01	90	90	90
l (210)	56 18	90	56 38′	90
m (110)	37 11	90	37 13	90
n ($\bar{2}12$)	31 33	29 36	31 16	29 38

Vrba‡ figures a crystal from Bosnia with the same predominating forms.

* This Journ., xlii, 403, 1891. † This Journ., xxi, 219, 1881, and letter.
‡ Zeitschr. f. Kryst., xv, 46, fig. 10.

(5) *Vesuvianite from New Mexico.*

Mr. W. M. Lippitt sent to me a single crystal with square cross section 5/8 inch on a side, which is apparently vesuvianite. It was found near the surface in a low range of hills near Bear Mountain, a little west of Pinos Altos, Grant Co., New Mexico. It is corroded and dull, but with the hand goniometer gives the angles of vesuvianite as closely as could be expected. The form is the unit pyramid (111) with mere suggestions of the prism. The specific gravity is low, 3·12.

(6) *Chrysoberyl Crystal from New York City.*

Mr. W. G. Levison submitted to me for measurement a crystal of chrysoberyl which he found, in 1893, in the rock taken from an excavation on 88th Street, near Amsterdam Avenue.

It is essentially the type of heart-shaped crystal with prominent pyramid *o* (111) described by Cathrein* as occurring at the emerald mines of Tokowaia, and is composed of two individuals with the usual twinning plane ρ (031). It is not tabular, the dimensions in the directions of the axes $\breve{a}$, $\bar{b}$, $\dot{c}$ being approximately $8\times10\times14^{mm}$. In color it is closely the green of the chrysoberyl from Haddam, Conn., and Petersdorf, Moravia.

The observed forms were:

Domes and pinacoids.—*a* (100), striated and yielding double images; *b* (010), dull; and *i* (011), large but very dull.

Pyramids.—*o* (111), bright and prominent on both individuals; *n* (121), small and yielding only a shimmer; *x* (515), doubtful, yielding a faint image between *o* and o''', but as *o* and o''' nearly coincide, it may be n''' of the second individual, which has essentially the same position.

Prisms.—*s* (120), bright and prominent, and *m* (110), occurring only, as indicated in the figure, in the deep striations of the faces *a*.

Angle.	Measured.	Calculated.	Error.
oo''	40° 11′	40° 7′	+4′
ba	90 4	90	+4
bo	70 01	69 56	+5
bn	53 58	53 51	+7
bs	46 52	46 46	−6
bm	64 57	64 50	+7
bi	60 16	59 53	+23
os	39 47	39 57	−10
ox	16 03	15 53	+10

* Zeitschr. f. Kryst., vi, 259, especially fig. 4.

The measured angles are sufficiently close to determine the forms, but are injured by the many dull faces and the difficulty of maintaining the rather heavy crystal and attached quartz in position.

The crystal was one of a cluster of three or four, as fragments of two others are visible as well as their cavities in the quartz gangue. The gangue also contains oligoclase, and attached to the sides of the cavities in the quartz are fragments of pyrite as if this mineral had filtered in between the chrysoberyl crystals and the quartz.

The crystal is shown in figure 5, the white portions shown in the faces *a* representing very nearly as in the actual crystal the deep grooves which yielded the signals corresponding to *m* (110).

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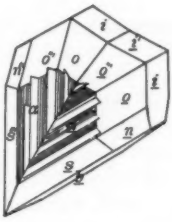

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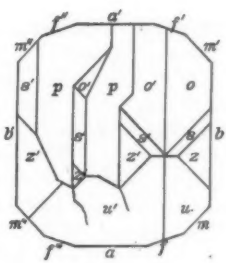

(7) *A Pyroxene Crystal from the Copper Mines of Ducktown, Tenn.*

Pyroxene crystals are recorded as occurring at the Ducktown Mines, but I have found no description of their faces, and for this reason and also for certain peculiarities I have thought this crystal worthy of description.

Among the specimens in the Egleston Mineralogical Museum is a rather large brown crystal of varnish-like luster and marked translucency. It measures in the directions of the axes $\breve{a}$, $\bar{b}$, $\dot{c}$ approximately $25 \times 35 \times 65^{mm}$. The gangue includes a wedge-shaped fragment of chalcopyrite 5^{mm} thick and 45^{mm} long attached to the vertical faces and a small amount of zoisite and fibrous mineral apparently amphibole.

The peculiarity of the crystal is, that while the prismatic portion belongs to a single individual with faces *a* (100), *b* (010), *m* (110) and *f* (310), the terminal faces indicate three individuals with a progressive change in faces, as shown in orthographic projection in figure 6, in which one face, u' ($1\bar{1}1$), is common to all three, showing only partially the lines of junc-

tion, the other faces belonging definitely to three individuals. Individual I, at the left, shows the well-developed faces z' ($0\bar{2}1$), s' ($\bar{1}\bar{1}1$) and p ($\bar{1}01$). Individual II, central, is a step above individual I and shows the combination of faces a little farther from the crystal center, and also a slight development of o' ($\bar{2}\bar{2}1$). Individual III, to the right, is composed of faces still farther from the center and is essentially complete, showing u, z, s, o, as well as u', z', s', o', but does not show p, which is the most prominent plane on I and II.

It would appear that during the growth of the crystal there was some change in conditions which resulted in a partial replacement of the comparatively smooth plane p ($\bar{1}01$) by the deeply corroded faces of o' ($\bar{2}21$).

Measurements were only practicable with the hand goniometer on account of the weight of the crystal.

	Measured.	Recorded.		Measured.	Recorded.
pb	91°	90°	*ff'*	37°	38° 36′
ob	106	105	*pa'*	74+	74 30
zb	40	41 25′	*oa'*	60+	61 32
sb	59	60 25	*za*	77	79 36
uu'	49	48 29	*sa'*	78	76 34
mm'''	93	92 50	*ua*	54	53 58

Columbia University, May 1901.

ART. XI.—*On the Motion of Compressible Fluids;* by J. WOODBRIDGE DAVIS.

IN the case of steady motion of a perfect fluid, the Eulerian differential equations of motion can be reduced to one equation in which the variation takes place along a stream-line. The integral of this is

$$\int \frac{dp}{\rho} = -\Omega - \frac{q^2}{2} + C.^* \qquad (1)$$

Applied to each elemental tube of flow, that is, to a tube of variable infinitesimal thickness whose boundary is composed of stream-lines, the equation of continuity becomes

$$\sigma q \rho = M.^\dagger \qquad (2)$$

p, ρ, q, σ, Ω, are the pressure, density, velocity, area of cross-section, and potential of extraneous forces per unit mass, at the variable distance s measured along the tube; C is the constant total energy per unit mass carried along the tube; M is the constant quantity of fluid that passes any section in unit time. ρ, σ, M, are necessarily positive in a physical problem: we may always assume that the direction of the motion is positive; and this we have done in forming equation (2). Hence q also will always be positive, if the motion is physically real.

When the fluid is compressible, and the pressure is a single-valued function of density only, such that, for all positive values of density, the density increases with increase of pressure, and diminishes with relief of pressure, and the ratio of the increment of pressure to the increment of density either remains constant or increases with compression, we may write

$$p = f(\rho), \frac{dp}{d\rho} \text{ is positive}; \frac{d^2p}{d\rho^2} \text{ is zero, or is positive.} \qquad (3)$$

From (1), (3),

$$\rho = \phi \left\{ \int \frac{dp}{\rho} \right\} = \phi \left\{ C - \Omega - \frac{q^2}{2} \right\} \qquad (4)$$

From (2), (4),

$$q.\phi \left\{ C - \Omega - \frac{q^2}{2} \right\} = \frac{M}{\sigma} \qquad (5)$$

σ is a single-valued function of s; and so is Ω for the forces of nature. But q is apparently not single-valued. At any point, it is to be found in terms of σ and Ω, by the solution of equation (5), which is equivalent to finding the inter-

* Lamb's Hydrodynamics, p. 23, eq. (3).

† L. c., p. 23, Art. 23.

sections of the curves represented by (2), (4), when σ, Ω, are constants, and ρ, q, are variables. (2) becomes the common hyperbola, which, as σ increases or diminishes, approaches or recedes from the intersection of the axes of ρ and q. One branch of this hyperbola occupies the quadrant of positive or physical values. To this quadrant we shall confine our attention. The slopes of the curves (4), (2), with respect to the

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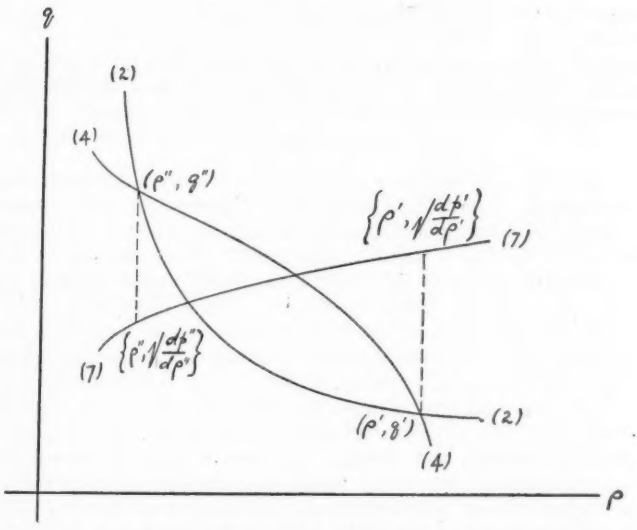

axis of ρ as a horizontal axis, that of (4) being determined from the second and third members of the equation, are

$$\frac{dq}{d\rho} = -\frac{1}{q\rho}\cdot\frac{dp}{d\rho}, \quad \frac{dq}{d\rho} = -\frac{q}{\rho} \tag{6}$$

Each curve continuously slopes downward from the axis of q to the axis of ρ. The line

$$q = +\left\{\frac{dp}{d\rho}\right\}^{\frac{1}{2}}, \tag{7}$$

according to (3), is either parallel to the axis of ρ or continuously slopes upward; hence it necessarily intersects (2). When (7) is parallel to the axis of ρ, dp is the product of $d\rho$ and a positive constant, the first member of (1) contains the logarithm of ρ as a factor, q in (1) or (4) is infinite where ρ is zero, and (4) reaches the axis of q at an infinite distance; hence (7)

intersects (4). When (7) has an intercept on the axis of q, one term of the first member of (1) is the logarithm of ρ multiplied by a positive constant, and (4) reaches the axis of q at an infinite distance; hence (7) intersects (4). When (7) passes through the origin, it necessarily intersects (4). Therefore, (7) always intersects (4) and (2), each in one point only.

If (4) and (2) intersect, the downward slope of the former is to the downward slope of the latter at the point of intersection according to the following ratio derived from (6):

$$\left[-\frac{dq}{d\rho}\right]_{(4)} : \left[-\frac{dq}{d\rho}\right]_{(2)} :: \frac{dp}{d\rho} : q^2. \qquad (8)$$

At an intersection that occurs below the line (7), the downward slope of (4) is greater than the downward slope of (2); hence there cannot be more than one intersection below the line (7). Likewise, above the line (7) cannot be more than one intersection, and there the downward slope of (4) is less than the downward slope of (2). Consequently, (4) and (2) cannot intersect at more than two points.

By variation of σ, (2) can be caused to intersect (7) at every point of the latter line, and consequently at the point where (4) crosses it. (4) and (2) are here tangent to each other, as indicated in (7) and (8); and the second derivatives reduced by (7),

$$\frac{d^2q}{d\rho^2} = -\frac{1}{q\rho}\cdot\frac{d^2p}{d\rho^2}, \qquad \frac{d^2q}{d\rho^2} = +\frac{2q}{\rho^2}, \qquad (9)$$

show that the contact is of the first order, and that in the immediate neighborhood of the tangent point the line (4) is between (2) and the coördinate axes. There can now be no points of intersection of (4) and (2), since at least one of the number could not conform to the law governing the slopes of the curves at the intersection, and there can be no other tangency, because a tangency cannot occur away from the line (7). So every point of (4), except the point of tangency, is between (2) and the coördinate axes. As σ is increased without limit, the tangency becomes two intersections, and these continue to exist until the whole length of (4) is traversed to the coördinate axes, because neither of the intersections can disappear without changing into a tangency, and no tangency can occur away from the line (7). The curves (4) and (2) have, therefore, two intersections, or one tangency, or two imaginary intersections; and q in (5), if it has one positive value, has also one other positive value, and only one other.

Let q', q'', denote the alternative velocities at any point of the tube. (2) and (3) show that there are two corresponding alternative densities, ρ', ρ'', and two corresponding alternative

pressures, p', p''. Of these two sets of values only one can represent the actual motion of the fluid; let this be q', ρ', p'. Necessarily q', ρ' are everywhere positive, according to the first paragraph, since the motion is actual; consequently q'', ρ'' are everywhere positive, according to the preceding paragraph. Hence the motion represented by q'', ρ'', p'', which may be designated the conjugate motion, is physically real, and is connected with the actual motion by the relation, contained in (1), (2), that in each case the same quantity of energy, CM, and the same quantity of matter, M, pass any section in unit time.

Pass now to a consecutive tube. The parameters and variables of the actual motion, as contained in equations (1), (2), (3), suffer an infinitesimal variation in this passage, one factor of the variation being the distance between the tubes. As this distance is brought to its limiting value, zero, the variation in p' becomes zero, so that the normal pressures on oppo site sides of a stream-line at every point are equal. That this condition of equilibrium exists is necessary to the hypothesis that there is an actual flow along these stream-lines. But the conjugate flow has the same parameters as the actual flow, and, therefore, in the passage from tube to tube, also suffers an infinitesimal variation that vanishes at a stream-line. Consequently, the conjugate flow is in equilibrium along the same stream-lines. It thus appears that any actual case of steady motion of a perfect compressible fluid whose compressibility is defined in expression (3), is one of two physically real solutions of the equations; and that the motions represented by these solutions differ in velocity at every point, but have the same stream-lines and boundaries, and discharge in unit time across any fixed mathematical surface the same quantity of matter and the same quantity of energy.

Differentiate (4) with respect to s, for the moment designating the quantity after ϕ by r:

$$\frac{d\rho}{dr}\frac{dr}{ds} = \phi' \left\{ \int \frac{dp}{\rho} \right\} \frac{1}{\rho}\frac{dp}{ds} = \phi' \left\{ C - \Omega - \frac{q^2}{2} \right\} \left\{ -\frac{d\Omega}{ds} - q\frac{dq}{ds} \right\}. \quad (10)$$

Divide by the differential equation of (1) with respect to s; there results

$$\phi' \left\{ \int \frac{dp}{\rho} \right\} = \phi' \left\{ C - \Omega - \frac{q^2}{2} \right\} = \phi' =, \text{ by (10)}, \frac{\rho d\rho}{dp}. \quad (11)$$

Now differentiate (5), using the several values of ρ, $d\rho \div ds$, and ϕ' in (4), (10), and (11) as convenience directs:

$$\rho\frac{dq}{ds} + q\frac{\rho d\rho}{dp}\left\{ -\frac{d\Omega}{ds} - q\frac{dq}{ds} \right\} = -\frac{q\rho}{\sigma}\frac{d\sigma}{ds}, \quad (12)$$

$$\frac{dq}{ds} = \frac{q}{q^2 - \frac{dp}{d\rho}} \left\{ \frac{dp}{d\rho} \frac{1}{\sigma} \frac{d\sigma}{ds} - \frac{d\Omega}{ds} \right\}. \qquad (13)$$

Let q' be the lesser of the alternative velocities, and q'' the greater. It has already appeared, as illustrated in figure 1, that q' is less than $\sqrt{dp' \div d\rho'}$, and that q'' is greater than $\sqrt{dp'' \div d\rho''}$. $\sqrt{dp \div d\rho}$ is the velocity of sound in the fluid where the density is ρ. It will be simpler and more intelligible to consider separately the effects of changes in σ and Ω; that is, to treat one as a constant while the other varies. Thus we learn from (13) that where the tube is narrowing, or the potential increasing, q' increases, and q'' diminishes; that where the tube is widening or the potential diminishing, q' decreases and q'' increases; and that where the tube is uniform in section, or the potential constant, q' and q'' do not change. Comparison of this statement with figure 1 shows, moreover, that the curve (4) approaches the intersection of the axes while Ω increases, and recedes from this point while Ω diminishes.

When q', q'', become equal, each satisfies equation (7). If this happens where the tube is narrowing, or the potential increasing, $dq \div ds = \pm \infty$, the upper sign pertaining to q' and the lower to q''. Represented graphically, with distances laid off as abscissæ and velocities as ordinates, the meeting of q', q'' occurs at a vertex having its convexity on the positive side. Beyond the vertex extend imaginary values to the section where appears a vertex with its convexity on the negative side. This must be where the tube is widening, or potential decreasing, and the velocity of the fluid is equal to the velocity of sound.

Equation (5) shows that wherever σ has the same value in the tube, extraneous forces not acting, or wherever Ω is the same in a tube of uniform cross-section, the values of q are the same. Hence the alternative velocities become equal, in the one case, wherever in the tube σ has a particular value, and, in the other case, wherever Ω has a particular value; and, wherever σ is less, or Ω is greater, than its particular value, the curves (4), (2), have receded from each other, and the velocities are imaginary; and, wherever σ is greater, or Ω is less, than the particular value, the curves (4), (2), have advanced upon each other, and the velocities are unequal, and become more and more separated as σ or Ω departs from its particular value.

While we confine our attention to a particular cross-section of the tube, suppose the quantity of matter, M, that in unit time flows across any section, retaining the constant energy C per unit mass, continuously increases from zero, as in an

infinite number of consecutive cases of steady motion. σ and Ω are now constant; the curve (4) is fixed, and the curve (2) recedes from coincidence with the axes to an indefinite distance. The alternative velocities approach each other, become equal, and become imaginary. Each section has, therefore, a limited capacity for the discharge of fluid. Evidently this capacity varies as σ, and is greater as the curve (4) is farther from the intersection of the axes, or as Ω is less, or as C is greater, and is less as Ω is greater or C is less. If the energy per unit mass should also increase with sufficient rapidity, the capacity of each section would not be reached; otherwise, it is ultimately reached, and the section whose area is least, or at which the potential is greatest, limits the capacity of the tube as a conduit.

When the capacity of the tube is not reached, the velocity is either everywhere less than the velocity of sound, or everywhere greater. When the capacity of the tube is reached, the velocity at the section of least area or where the potential is greatest, is equal to the velocity of sound at that point, and there the first derivative of the velocity with respect to distance assumes the indeterminate form $\frac{0}{0}$. To resolve this, differentiate (2) with respect to s, and cancel the term containing $d\sigma \div ds$; then

$$\frac{d}{ds}\left\{\frac{dp}{d\rho}\right\} = \frac{d^2p}{d\rho^2}\frac{d\rho}{ds} = -\frac{d^2p}{d\rho^2}\frac{\rho}{q}\frac{dq}{ds}. \qquad (14)$$

With the aid of (14) differentiate numerator and denominator of (13) once and solve for $\frac{dq}{ds}$:

$$\frac{dq}{ds} = \pm\sqrt{\frac{\frac{dp}{d\rho}\frac{q}{\sigma}\frac{d^2\sigma}{ds^2}}{2q + \frac{d^2p}{d\rho^2}\frac{\rho}{q}}}, \text{ or } \pm\sqrt{\frac{-q\frac{d^2\Omega}{ds^2}}{2q + \frac{d^2p}{d\rho^2}\frac{\rho}{q}}}. \qquad (15)$$

When $\frac{d^2\sigma}{ds^2}$, or $\frac{d^2\Omega}{ds^2}$, is zero, $\frac{dq}{ds} = \pm 0$.

When the capacity of the tube is reached, the conjugate flows meet at the critical section with equal velocities, densities, and pressures, and are, consequently, there indistinguishable. On either side of this section, which of the two possible flows is actual, is determined by conditions existing on that side. Hence it is possible for the actual flow to have a velocity less than sound on one side and greater on the other. That this is not a natural case appears as follows: On either side of

the critical section the density in one of the flows is everywhere considerably greater than in the other, and, therefore, the mass of fluid is considerably different in the two flows. Now suppose the quantity, M, of fluid passing any section in unit time increases from zero to the capacity of the critical section. Through all this range the velocity of the fluid everywhere in the tube remains on one side of the velocity of sound until at the limit it may still remain everywhere on that side, or may pass from one side to the other at the critical section. Hence one solution at the limit is continuous with all the previous solutions, and the other is isolated. The isolated case cannot originate from a flow that increases to the capacity of the tube, for this would involve at the final instant in the total mass of the fluid an instantaneous change of considerable magnitude. Neither can it exist in a tube that returns into itself and has only one section of least area or of greatest potential.

The behavior of the conjugate flows in any space may be summed up as follows: If in either flow the velocity of the fluid at any point is less than the velocity of sound at that point, then, in general, the velocity of the fluid is everywhere less than the velocity of sound, or may reach the latter as a limit. If the velocity is greater than that of sound at one place, it is greater at all places, or may diminish to the velocity of sound as a limit. One conjugate velocity is everywhere greater than the other conjugate velocity, or may diminish to equivalence with the latter as a limit where each becomes equal to the velocity of sound. The places of greatest velocity and least density in one case are the places of least velocity and greatest density in the other. In the approach to a gorge or place of higher potential the more rapid flow is retarded and the other is accelerated; the recession from a gorge or place of higher potential produces inverse effects. Under very special conditions the velocity of the fluid is not limited by the velocity of sound.

There is an inferior limit, ρ_0, to the density of a compressible liquid, whereas the density in the pressure-density formula, (3), of that liquid, and, therefore, in equation (4), is continuous from zero to infinity. Hence the foregoing conclusions have a limited application to liquids. Suppose the line $\rho = \rho_0$ to be drawn in figure 1. If now the actual motion is such that both intersections of (4) and (2) always occur on the positive side of the line $\rho = \rho_0$ for every elemental tube of flow, there is also a conjugate motion; otherwise, there is no conjugate motion, and the conclusions of the preceding paragraph are only partially applicable.

NOTE. On page 177, volume x of the Memoirs of the Manchester Literary and Philosophical Society, 1887, in a paragraph of a paper entitled "On the Flow of Gases," read before the Society November 17, 1885, Professor Osborne Reynolds notes that, when a gas is flowing through a channel of variable section, the pressure at every point of the channel may have in general either of two unequal values; whereas, if the fluid were incompressible, the pressure could have but one value. This is the only published notice the present writer has seen of the dual values that occur in problems relating to the motion of compressible fluids. The apparent governing effect of the velocity of sound upon the behavior of compressible fluids, is set forth in an interesting manner by Professor Reynolds in the paper just mentioned, and by Hugoniot in a communication published in Comptes Rendus, page 1178, volume ciii, 1886, in the case considered by these authors of a flow through an orifice or minimum section. The present writer has recently been perplexed by meeting with the dual values in several problems. Professor Horace Lamb of Manchester, England, who courteously examined one of the problems, and rendered the solution more definite by employing the method of discussing the intersections of the curves (2), (4), as followed in this paper, expressed the decided opinion that only those values which are continuous with the observed or assumed values at a given point belong to the problem in hand. Impressed by this statement, the writer set about inquiring into the meaning of the values that appear, only to be disregarded, with the result, arrived at in the foregoing paper, that they belong to a separate motion related to the actual motion, as the branches of a hyperbola are related, by having the same equations and the same parameters.

ART. XII.—*The Action of Sodium Thiosulphate on Solutions of Metallic Salts at High Temperatures and Pressures;* by JOHN T. NORTON, JR.

[Contributions from the Kent Chemical Laboratory of Yale University—C.]

THE use of sodium thiosulphate as a substitute for hydrogen sulphide in effecting precipitations and its application in the case of arsenic, antimony, copper and platinum was suggested by Himly* before the middle of the present century. Thirteen years later Vohl† and Slater,‡ independently, drew attention to this use of sodium thiosulphate and extended the investigation to salts of tin, mercury, silver, gold, lead, bismuth and cadmium. Slater in addition studied the action of sodium thiosulphate upon chromic acid, molybdates, ferrous and ferric ferrocyanides, ferric sulphocyanide and potassium permanganate. Following out these lines, the precipitation of copper together with arsenic and antimony by treating with sodium thiosulphate the hot solution containing sulphuric acid and the separation of these elements from tin, zinc, iron, nickel, cobalt and manganese has been advocated by Westmoreland§; and quite recently Faktor‖ has studied the action of sodium thiosulphate upon neutral salts of several of the elements mentioned, as well as the modifying influence of ammonium chloride and other salts upon the course of the reaction.

Subsequently to the work of Himly, Vohl, and Slater, Chancel¶ developed his well-known method for the precipitation of aluminum as the hydroxide and its separation from salts of iron by boiling with sodium thiosulphate the nearly neutral solution, containing the salts of aluminum and iron, at suitable dilution; and upon an extension of the principle of Chancel's separation of aluminum from iron, Stromeyer** founded his well-known processes for the separation of titanium and zirconium from iron. The latter process appears to be fairly trustworthy; but of Chancel's method, although it has met with wide acceptance, it was shown by Wolcott Gibbs very soon after its announcement†† that it fails to bring about complete separation of alumina within a reasonable period of boiling, and this result has been confirmed by Zimmerman,‡‡ who has shown that the boiling must be continued fifteen hours in order to complete the precipitation of the alumina.

* Ann. Chem. (Liebig), xliii, 150.
‡ Chemical Gazette, 1855, p. 369.
‖ Centralblatt, 1900, ii, 20, 67, 239, 594.
** Ann. Chem. (Liebig), cxiii, 127.
‡‡ Inaug. Diss., Berlin, 1887.
† Ann. Chem. (Liebig), xcvi, 237.
§ Jour. Soc. Chem. Ind., v., 51.
¶ Compt. rend., xlvi, 987.
†† Zeitschr. anal. Chem., iii, 389.

It was shown by Dr. Gibbs that when the treatment of salts of aluminum by thiosulphate was carried on in sealed tubes under pressure at 120° C., the precipitation of alumina was complete, and further that the precipitation of sulphides of nickel, cobalt and iron, though partial under ordinary atmospheric pressure, was made complete by heating in sealed tubes to 120°–140° C.

In repeating the experiments of Dr. Gibbs qualitatively and extending them, I have made use of the well-known Pfungst tube to secure the necessary pressure. In each experiment a test tube containing the mixture of an excess of sodium thiosulphate with the salt whose action was studied was placed within the Pfungst tube containing some water, the cover of the latter was set in place and firmly bolted upon a washer of lead, and the whole was submitted to temperatures varying from 140° to 200° C. for an hour by immersing in a bath of paraffine. After cooling, the test tube was taken out, the precipitate was filtered off, and the filtrate tested by appropriate reagents to determine the completeness of precipitation. The following table records the details of these experiments:

ACTION OF $Na_2S_2O_3$ ON SALTS UNDER PRESSURE.

Salts used.	Precipitates.	Degree of Precipitation.
	Sulphides.	
$NiSO_4$	$NiS + S$	Complete.
$CoSO_4$	$CoS + S$	"
$FeCl_3$	$FeS + S$	"
$ZnSO_4$	$ZnS + S$	"
$PbO_2(C_2H_3O)_2$	$PbS + S$	"
$Hg(NO_3)_2$	$HgS + S$	"
$AgNO_3$	$Ag_2S + S$	"
$CuSO_4$	$CuS,(Cu_2S), + S$	"
$CdSO_4$	$CdS + S$	"
$KSbC_4H_2O_7$	$Sb_2S_3 + S$	"
$Bi(NO_3)_3$	$Bi_2S_3 + S$	"
	Hydroxides.	
$NH_4Al(SO_4)_212H_2O$	$AlO_3H_3 + S$	"
$K_2Cr_2O_7$	$CrO_3H_3 + S$	"
K_2ZrF_6	$ZrO_4H_4 + S$	"
K_2TiF_6	$TiO_4H_4 + S$	"
$Th(NO_3)_4$	$ThO_4H_4 + S$	"
	Elements.	
SeO_3	$Se + S$	"
TeO_3	$Te + S$	"

Salts used.	Precipitates.	Degree of Precipitation.
	Sulphides.	
$MnSO_4$	$MnS + S$	Partial.
$AuCl_3$	$Au_2S + S$	"
$(NH_4)_2MoO_4$	$MoS_3(?) + S$, Red liquid	"
	Hydroxides.	
$BeCl_2$	BeO_2H_2	"
	Undetermined.	
$(NH_4)_2U_2O_7$	Black	"
K_2PtCl_6	Gray, reddish brown liquid	"
$CeCl_3$	White, yellow liquid	"
$CaCl_2$	" " "	"
$SrCl_2$	" " "	"
$BaCl_2$	" " "	"
$MgSO_4$	———	Trace.
NH_4VO_3	Brown liquid	"
H_2KAsO_4	———	"

A perusal of this table brings to light several interesting facts. It appears that salts of nickel, cobalt, iron, zinc, lead, mercury, silver, copper, cadmium, antimony and bismuth are completely precipitated as sulphides by sodium thiosulphate under the prevailing conditions of temperature and pressure. In the case of manganese precipitation is only partial, and arsenic does not seem to be precipitated from an arsenate without the addition of acid. Tin, curiously enough, is not thrown down as the sulphide from a stannous salt, but gives a dirty white precipitate of uncertain composition. Salts of aluminum, chromium, titanium, zirconium and thorium are completely precipitated as the hydroxides; but in the case of beryllium, which one would expect to act similarly, the precipitation as the hydroxide is incomplete. Salts of selenium and tellurium are reduced, and the elements are precipitated. The precipitates obtained with barium, strontium and calcium were white in a bright yellow liquid, but no study was made of the constitution of precipitate or liquid. In the case of magnesium there was no precipitate. Salts of molybdenum, vanadium and uranium gave dark-colored liquids. Thallium yielded a white spongy mass which on compression was reduced to a very small bulk without disintegrating. Salts of gold and platinum gave slight dark precipitates, presumably sulphides, surrounded by dark-colored liquids.

The apparatus used in these experiments and described above is easily handled and answers sufficiently well for qualitative purposes. But, obviously, the introduction into precipitates of foreign matter caused by the action of water on

the glass of the test-tube and porcelain lining of the Pfungst tube precludes the possibility of an exact quantitative study of the reactions involved. For the subsequent experiments, therefore, conducted upon the same general lines, a digester with an interior cylindrical cavity of about 12^{cm} in depth by 5^{cm} in diameter, and provided with a pressure gauge, was employed. As a container for the solutions to be tested use was made of a platinum cylinder, 4^{cm} in diameter and 10^{cm} deep, provided with a loose cover. With this apparatus the following quantitative experiments, which deal with those elements which are precipitated as hydroxides, namely, aluminum, beryllium, chromium, zirconium and titanium, were made.

In each case a weighed quantity of the salt taken for the experiment was dissolved in 50^{cm} of water in the platinum vessel, and to this a known amount of sodium thiosulphate was added. The vessel was placed in the digester, and the latter was heated by a Bunsen burner in the customary way until the required pressure was shown on the gauge. The apparatus was then cooled and the platinum vessel removed from the digester. The precipitate was filtered off on ashless paper, ignited, and weighed.

Experiments with a Salt of Aluminum.

In a series of experiments made according to the method of Chancel, the results of which are shown in Table II, the solution in water of a weighed portion of pure ammonium alum was treated with an excess of sodium thiosulphate and boiled vigorously for periods varying from ten minutes to half an hour.

TABLE II.

	Amount of Alum taken as Al_2O_3. grms.	Amount of $Na_2S_2O_3$. grms.	Al_2O_3 found. grms.	Error. grms.
1.	·0537	Large excess	·0471	·0066—
2.	·0537	" "	·0397	·0140—
3.	·1083	" "	·0931	·0152—
4.	·1137	5 grms.	·0979	·0158—
5.	·1139	2 "	·1002	·0137—

These results substantiate the observations of Gibbs* and of Zimmerman,† and show clearly that the boiling of solutions of the aluminum salt and sodium thiosulphate for a reasonable time does not effect the complete precipitation of aluminum as the hydroxide.

Table III shows the result of submitting solutions of ammonium alum treated with varying quantities of sodium thio-

* Loc. cit. † Loc. cit.

sulphate to a pressure of 20 atmospheres in the digester. It usually required about 40 minutes to raise the pressure to the limit set; but this limit once reached, the digester was allowed to cool slowly. The duration of an experiment was about two hours.

TABLE III.

Alum taken as Al_2O_3. grms.	Amount of $Na_2S_2O_3$ used. grms.	Al_2O_3 found. grms.	Error. grms.
·0565	5	·0633	·0068 +
·1132	10	·1154	·0022 +
·1153	5	·1186	·0033 +
·1128	3	·1129	·0001 +
·1126	3	·1142	·0016 +
·1128	2	·1120	·0008 —
·1136	2	·1121	·0015 —
·1128	2·5	·1136	·0008 +
·1124	2·5	·1127	·0003 +
·1134	2·25	·1133	·0001 —

This table shows that sodium thiosulphate precipitates aluminum completely as the hydroxide when pressure is employed. The high results seen in some of the experiments appear to be due to the difficulty of removing by ignition the large amounts of sulphur formed in the action, as well as to the salts mechanically included in the precipitate. The amounts of sulphur and contaminating salts present depend upon the amount of thiosulphate taken; therefore this should be as small as possible, 2 to 3 grms. being sufficient to precipitate all the alumina in a gram of alum. When the amount of thiosulphate is reasonably restricted the weights of alumina accord fairly well with the theory.

Experiments with a Salt of Chromium.

Up to the time of the completion of this work nothing appears to have been done upon the quantitative precipitation of chromium as the hydroxide by means of sodium thiosulphate. Slater* and Rose† make mention of the action of sodium thiosulphate upon chromic acid, bichromates and neutral chromates, but give no quantitative data. Recently, however, F. Faktor‡ has studied the action of sodium thiosulphate on chromium compounds. This investigator has found that if aqueous solutions of potassium bichromate and sodium thiosulphate are boiled together, a brown precipitate of hydrated Cr_2O_3, CrO_3 separates out and the liquid turns yellow owing to

* Loc. cit. † Traite de Chimie Analytique, vol. i, p. 479.
‡ Zeitschr. anal. Chem.. 1900, xxxix, 345.

the formation of normal chromate. A solution of potassium chromate is unaffected by boiling with thiosulphate, but in presence of ammonium or of magnesium chloride the chromium is separated rapidly and completely in the same form as with the bichromate, and after continued boiling with an excess of thiosulphate all the chromium present is precipitated. Faktor also found that a solution of chromic chloride is completely decomposed by continued boiling with thiosulphate, chromic hydroxide and sulphur being precipitated.

In the experiments shown in Table IV a weighed quantity of pure potassium bichromate was dissolved in water, a known amount of sodium thiosulphate added, and the whole submitted to a pressume of 20 atmospheres in the digester. After cooling, the precipitate was filtered off on an ashless paper, ignited and weighed as Cr_2O_3.

TABLE IV.

	$K_2Cr_2O_7$ taken as Cr_2O_3. grms.	Amount of $Na_2S_2O_3$. grms.	Cr_2O_3 found. grms.	Error. grms.
1.	·1330	3	·1341	·0011 +
2.	·1330	2.5	·1326	·0004 −
3.	·1322	2·5	·1318	·0004 +
4.	·1303	2	·1303	·0000 —
5.	·1301	2	·1310	·0009 +
6.	·1320	2	·1322	·0002 +

The results of these experiments are very satisfactory and show that under pressure sodium thiosulphate precipitates chromium rapidly and completely as the hydroxide. It is advisable to use as small a quantity of thiosulphate as possible in order to prevent the presence of much free sulphur in the precipitate.

Experiments with a Salt of Beryllium.

In experiments dealing with beryllium the salt used was the chloride, a certain amount of which was dissolved in water diluted to a liter and the amount of beryllium present determined by precipitating with ammonia and weighing as the oxide. Measured quantities of this solution were drawn from a burette as required. When a solution of a salt of beryllium and sodium thiosulphate are merely boiled together, nearly all the beryllium remains in solution. It was expected that the use of pressure would throw out all the beryllium, but, curiously enough, when solutions of beryllium chloride and sodium thiosulphate were submitted in the digester to pressures ranging from 10 to 80 atmospheres, only a partial precipitation of the hydroxide took place.

Experiments with Salts of Zirconium.

To prepare a standard solution of the salt of zirconium it was found to be most convenient to heat the double fluoride of potassium and zirconium with sulphuric acid, evaporate to dryness in platinum, dissolve the zirconium sulphate remaining in water and enough sulphuric acid to prevent the precipitation of the basic salt, and dilute to standard volume. Measured portions of the solution were taken from a burette as required for the experiments. The presence, however, of so large an amount of sulphuric acid as was necessary to keep the zirconium salt in solution tends to decompose sodium thiosulphate so rapidly that it was found necessary to nearly neutralize the solution with ammonium carbonate before adding the sodium thiosulphate. The solution of zirconium sulphate was standardized by precipitating with ammonia aud weighing as the oxide.

In experiment 1 of Table V, the solutions of zirconium sulphate and sodium thiosulphate were boiled together for a few minutes and then the precipitate filtered off, ignited and weighed as the oxide. In experiments 2–5 inclusive, similar solutions were submitted to a pressure of 20 atmospheres in the digester.

TABLE V.

	ZrO_2 taken. grms.	$Na_2S_2O_3$ taken. grms.	ZrO_2 found. grms.	Error. grms.
1.	·0658	3	·0651	·0007 −
2.	·0658	3	·0676	·0016 +
3.	·0666	2	·0670	·0004 −
4.	·0641	2	·0648	·0007 +
5.	·0641	2	·0645	·0004 +

These results clearly show that sodium thiosulphate precipitates zirconium completely as the hydroxide either with or without the aid of pressure.

Experiments with a Salt of Titanium.

The solution of the salt of titanium was obtained by treating the double fluoride of potassium and titanium with sulphuric acid, evaporating to dryness, and dissolving the residue in sulphuric acid and water. The solution was standardized by precipitating the titaninm hydroxide with ammonia and then adding an excess of acid as recommended by Gooch.* This method of procedure avoids the tendency to excessive weight observed when the titanium hydroxide is precipitated by ammonia in presence of salts of the alkalies.

* Amer. Chem. Jour., vii, 285.

In the following table is shown the effect of treating a solution of titanium sulphate with sodium thiosulphate. Experiment 1 was conducted by merely boiling a solution of the reagents named above, filtering off the precipitate and weighing as the oxide. In experiments 2 and 3 the solution of titanium sulphate and sodium thiosulphate was submitted to a pressure of 20 atmospheres in the digester.

TABLE VI.

	TiO_2 taken. grms.	Na_2S_2O taken. grms.	TiO_2 taken. grms.	Error. grms.
1.	·0240	2	·0237	·0003—
2.	·0240	2	·0240	·0000±
3.	·0240	2	·0240	·0000±

These results show that titanium is completely precipitated by sodium thiosulphate either with or without the aid of pressure.

To recapitulate: I have shown that sodium thiosulphate will completely precipitate aluminum, chromium, zirconium and titanium as the hydroxides with the aid of high temperature and pressure. Beryllium is only partially precipitated under similar conditions. Mere boiling for a reasonable time will not precipitate aluminum and chromium, but it is sufficient in the case of zirconium and titanium.

In conclusion I wish to thank Prof. F. A. Gooch for his kind advice and assistance.

ART. XIII. — *Secondary Undulations Shown by Recording Tide-gauges;* by A. W. DUFF.

ON recording tide-gauges at many ports rapid oscillations are observed crossing and recrossing the main tidal record. Mr. W. Bell Dawson* in a short article seems to have been the first in recent years to recall attention to these "secondary undulations," and well describes them as standing "in much the same relation to the main tidal wave as a higher octave would to a low musical note when their undulations are recorded graphically." Others have given occasional attention to them, especially Mr. F. Napier Denison,† whose two brief papers suffer somewhat from a lack of detailed evidence for the "ten chief points deduced" and too exclusive a concern with the meteorological aspects of the question. No one seems to have attempted to give a full account of the phenomena for even a limited region. The best field for the study of the question is apparently the Eastern coast of Canada bounded by the Bay of Fundy, the Atlantic Ocean and the Gulf of St. Lawrence, where, because of the great extent and complexity of the tides, numerous recording tide-gauges have been established in the interest of navigation.

Four years ago I published‡ some observations made at St. John, N. B. The evidence advanced, depending on a calculation of the period of undulation from the dimensions of the basins in the neighborhood, seemed to point strongly to the undulations being of the nature of free vibrations of a partially confined body of water under the force of gravity. Two years ago I took advantage of a visit to Ottawa, Canada, to examine (with the permission of the authorities of the Tidal Survey of the Dominion of Canada, to whom my thanks are due for their courtesy) the tidal records from the Canadian tidal stations and made extensive notes therefrom. Other occupations interfered with the pursuit of the subject, but as no one has since then taken it up, I have thought it well to publish a brief outline of the information gathered together with an examination of its bearing on certain proposed explanations.

Materials studied.

In order to make clear the extent of the data employed in what follows, it may be stated that the tidal records from twenty stations were examined in detail, these records covering

* Trans. Roy. Soc. Can., i, 1895-96.
† Proc. Can. Inst., Jan. 16, 1897; April 23, 1898.
‡ This Journal, iii, 406, 1897.

a period of seven years, although the records from many stations covered a much shorter period. In all, the records of 404 tides or parts of tides that presented evidence of secondary undulations were copied carefully on tracing paper. In addition, the files of the Atlantic Pilot Chart (monthly) and the Canada Weather Map (weekly), were examined and in many cases copied for periods corresponding to the tracings of tidal records. From the Admiralty Charts of the region the width and mean depth of many bays and channels were obtained. To facilitate the work of any future investigator, I shall, so far as possible, give the dates of occurrences noted (usually in foot notes), but I find that occasionally my notes of dates are somewhat imperfect.

The period, extent and general nature of the undulations.

In the following statements of observed periods of regular undulations, I have not, so far as I know, omitted any occasions on which regular undulations could be observed no matter what the period might seem to be (except in the cases of St. John and Halifax, for which the records are too extensive to be quoted in full). In reference to the fewness of these occasions at certain stations, it is to be remembered that at many places tide-gauges were maintained for a few months only.

1. St. John, N. B. (on the Bay of Fundy). Here the undulations are frequent, sometimes continue for as much as a week, and vary in extent from an inch to a foot; but, when very slight, they are hardly distinguishable owing to the great extent of the tide (at spring tides 28 feet). The undulations are more nearly regular and periodic than at any other station, the mean period (the evidence for which will be stated in another connection) being 41 minutes. Occasional variations of several minutes from the mean period are found, but slight irregularities at times make the estimation of the period difficult. The undulations occur about four times as frequently in winter as in summer.

2. Yarmouth, N. S. (on the Bay of Fundy). The undulations are very common especially near low tide when they are usually present, and in certain extreme cases (e. g. Jan. 1, 1899) nearly five feet in amplitude; at high tide they are much less marked. Frequently they are more or less irregular, whereas at other times (e. g. Dec. 31, 1898) as many as ten regular oscillations may be measured. From twelve records of low tide which showed regular undulations the following mean periods were deduced:* 28, 27, 34, 34, 28, 32, 26, 22, 31, 34, 28. 34,

* June 26 (5 undulations); July 2 (6), 5 (5), 6 (4); Aug. 30 (7); Sept. 8 (5), 11 (6), 18 (5); Oct. 12 (6), 16 (4), 30 (8), 30 (4); Dec. 31 (10)—all in 1898.

28 minutes—final mean 30m. At high tide the period is entirely different and fewer complete oscillations occur :* 65, 70, 70, 67, 70, 68, 75, 65, 72, 69, 65, 65, 70, 70m.—mean 69m. At times of violent disturbances during gales there is a tendency for oscillations of both periods to persist, along with a running accompaniment of very minute and very rapid oscillations.

3. Digby, N. S. (on Annapolis Basin connected with the Bay of Fundy by a narrow strait—Digby Gut). The undulations are always small, rarely exceeding an inch and therefore difficult to measure with accuracy. On the only occasions when the extent was sufficient the periods were :† 12, 11, 11, 11, 10, 11, 13—final mean 11m.

4. Westport, N. S. (on Bryer Island, Bay of Fundy, facing Grand Passage between Bryer Island and Long Island). Undulations are comparatively rare, of short period, and never exceeding 4 inches ; usually they are very irregular and have an appearance of two or more systems superposed. The gauge only ran for a few months and on only six occasions were there clear oscillations, and on only three occasions did steady regular oscillations seem to emerge from the confusion, the mean periods being‡ 17, 14, 16 m.—mean 15 m. (but not much reliance can be placed on this figure).

5. Welchpool, Campobello Island, N. B. (on a narrow winding channel flowing into the Bay of Fundy). Here there seem to be no secondary oscillations.

6. Hopewell Cape, N. B. (on Shepody Bay at head of Bay of Fundy). 7. Parrsboro, N. B. (on Minas Basin at the head of the Bay of Fundy). 8. Windsor, N. S. (on the Avon River which flows into Minas Basin). At the above three stations the range of the tide is, as is well known, very great and the tide-gauges only registered the crests of the tidal waves. No secondary undulations are shown.

The records for the two points next noted are not from Tidal Survey gauges but from a portable recording tide-gauge of my own construction. Accounts of these have already been published.§

9. St. John River, N. B. (The station of observation was on a small bay in the mouth of the river, separated from the harbor by the narrow gorge, only 100 yards wide, through which the river rushes into the harbor.) There are two series of oscillations, one of a period between thirty and forty min-

* Sept. 27 (1); Oct. 7 (1), 8 (2), 9 (2), 12 (1), 21 (2), 31 (2); Nov. 6 (2), 16 (2) 18 (1), 22 (2): Dec. 4 (1), 5 (1), 6 (1)—1898.

† July 1 (6), 18 (4); Aug. 13 (8), 27 (11); Oct. 24 (9), 27 (5); Nov. 27 (6)—1898.

‡ Oct. 13 (5); Dec. 26 (7), 27 (4)—1898.

§ This Journal, iii, 1897. Trans. Nat. Hist. Soc. of New Brunswick, 1897.

utes, apparently identical with the oscillations shown by the official gauge in the harbor, the other a series of very rapid oscillations having a period of only 35 *seconds*.

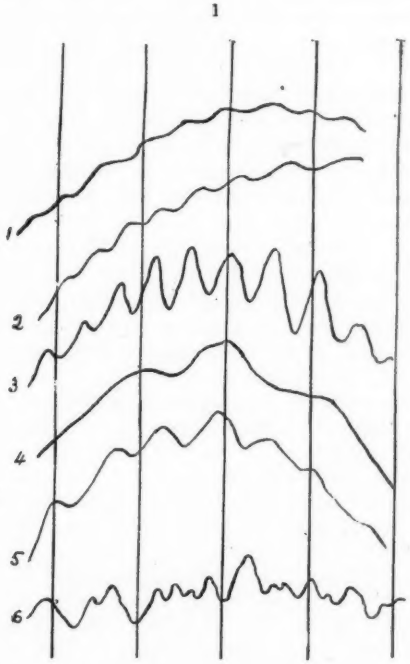

FIGURE 1.—Examples of regular secondary undulations. 1, Pictou; 2, Forteau Bay; 3, Yarmouth at low tide (examine with page inverted); 4, Yarmouth at high tide; 5, St. John; 6, Grindstone Island (as near an approach to regularity as could be found). The vertical scale is different for different curves.

10. Quaco, N. B. (about 20 miles farther up the Bay of Fundy than St. John). Three different records of secondary undulations were obtained, giving in all three cases a mean period of 12·5 m.

This completes the list of Bay of Fundy stations. Halifax is the only station not on either the Bay of Fundy or the Gulf of St. Lawrence, and may be next discussed.

Halifax, N. S. (on Halifax Harbor, Atlantic Coast of N. S.) Here secondary undulations are nearly always present through the whole range of the tide. Frequently they are very irregular and often of great amplitude, but to an eye accustomed to the curves representing the composition of simple harmonic

motions of different periods, the irregularity is at times markedly regular, showing the coexistence of at least two systems of undulations. As will be seen by fig. 2, the undulations at

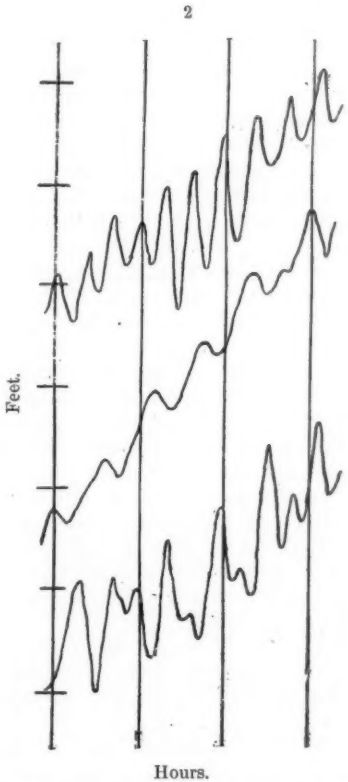

FIGURE 2.—Types of secondary undulations at Halifax, N. S.

times become simple and quite remarkably regular. At such times the period is either about 20 minutes or about 33 minutes. The figure shows both series and the apparent composition of the two in the third curve.

I pass now to a consideration of the records from stations on the Gulf of St. Lawrence and adjacent basins.

1. St. Paul's Island (in Cabot Strait between Newfoundland and Cape Breton, N. S. and connecting the Gulf of St. Lawrence with the Atlantic Ocean). The secondary undulations are here practically continuous and vary from fine shadings so

close together as to defy copying, to clear cases of a definite measurable period. Sometimes as many as 40 oscillations can be counted. The mean periods obtained are* 4·7 m., 4·8 m., 4·8 m., 4·65 m., 4·7 m., 4·7 m.—final mean 4·7 m. Along with these there occasionally occurs a series of much longer period depending apparently on a state of high wind. The period in such cases is difficult to determine accurately owing to the other markings referred to. The following exceptionally clear cases are of particular interest. On Oct. 15th, 1898, 11 regular oscillations gave a mean of 12·9 m. and on Oct. 16, 1898, 14 oscilations gave a mean of 12·9 m. In the latter case the amplitude was at times a foot and the whole tide was elevated 1½ feet both at high water and at low water (a marked sign of high wind).

2. Souris, Prince Edward Island. Secondary undulations are almost always present. The extent is usually only one or two inches, at times three inches, occasionally swelling out to seven inches and showing strong signs of the coexistence of systems of different period. The only cases† in which the periods were definite and determinable gave 22·5, 21·8, 21·2, 21·2, 22·0, 21·6, 20·0, 22·4, 24·5 minutes—mean 21·9 minutes.

3. St. Peter's Bay, P. E. I. Secondary undulations are very common but usually very irregular. At times the irregularity shows evidence of the superposition of different systems. On a very few occasions‡ the undulations become regular, giving mean periods of 19, 22, 21 m.—final mean 20 m. On another occasion§ seven very regular oscillations gave a mean period of 30 m. pointing probably to the conflicting system. The gauge was only a month in operation.

4. Charlottetown, P. E. I. (on an irregular shallow bay connected by a narrow entrance with Northumberland Strait between P. E. I. and N. B.). Secondary undulations very rare and irregular.

5. Pictou, N. S. (at the eastern end of Northumberland Strait). Secondary undulations are frequent and persistent but of small extent; periods‖ 33, 31, 26, 24, 30, 30, 28—mean 29 m. The preceding were from the records of a large gauge in the summer of 1896. In the following year a small gauge replaced the large one and periods¶ were observed of 26, 27, 32, 31 m.—mean 29 m. as before.

* July 4 (10), 3 (20); Oct. 3 (17), 1 (14); July 25 (40) Sept. 9 (7)—1898.

† July 8? (4), 17 (11); Aug. 17 (4), 24 (4), 29 (5), 29 (6); Sept. 20 (3), 19 (7), ? (4).

‡ Nov. 15 (5), 13 (4), 15 (10).

§ Nov. 16 (7).

‖ Sept. 7 (5); Oct. 27 (6); Nov. 1 (6); June 20 (4); June 27 (3); June 30 (3); July 6 (5)—1896. (Occasionally some doubt as to exact date.)

¶ July 7 (10); Sept. 21 (5), 23 (5), 24 (8).

6. Cape Tormentine, N. B. (on the narrowest part of Northumberland Strait). Apparently no secondary undulations.

7. Lower Neguac, N. B. (at mouth of Miramichi Bay, where the gauge was placed "to obtain the open tide unaffected by the bars and rivers of the bay"). No evidence at all of secondary undulations.

8. Carleton, Quebec (on Chaleur Bay, a branch of the Gulf). Secondary undulations occur about once a week, but of such small extent and duration that no period can be deduced.

9. South West Point, Anticosti. Secondary undulations are almost incessant and frequently very irregular, but at times clear, regular, well sustained undulations occur:* 18, 18, 18, 19, 20, 18, 18, 18, 21, 20, 21—mean 19 m.

10. Grindstone Island (of the Magdalen Island group in the middle of the Gulf). Secondary undulations are nearly incessant but quite irregular and no clear period is found. (The data are perhaps insufficient as the gauge was only a short time in action.)

11. Forteau Bay (on the Strait of Belle Isle, between Newfoundland and Labrador). Secondary undulations are almost incessant, and usually very irregular. Rarely does a clear period emerge†: 17, 15, 16—mean 16m.

Résumé.

The secondary undulations at any place are at times irregular (rarely at St. John), at other times regular (perhaps never at Grindstone Island); when they are regular the period of oscillation has a distinct and characteristic value for each place, varying from place to place, from less than a minute to over an hour. The amplitude also varies widely. At some places at least two systems of regular oscillations of different periods are found; these may exist together (Halifax), or the period at low tide may be distinct from the period at high tide (Yarmouth).

Theories advanced to explain Secondary Undulations.

It will be convenient at this point to summarize the theories that have been offered to account for the preceding phenomena. There are two chief points that require explanation, the *origin* of the initial disturbance and the particular *period* (or periods) at each place. These two points may be entirely distinct or the explanation of one may include that of the other.

* Because of some slight difficulty in ascertaining exact dates I noted number of tidal sheet:—238 (7), 238 (12), 241 (8), 249 (5), 249 (5), 256 (25), 261 (8), 262 (9), 271 (5), 271 (5), 274 (5).

† These all occurred on Oct. 30–31, 1898.

(1). It has been supposed that the results are due in reality to peculiarities in the action of each tide gauge.—This suggestion hardly deserves serious consideration, and is only included here because it is so admirably disposed of by the case of Pictou (stated above), where two entirely different gauges in different years showed secondary undulations of the same mean period.

(2). Earthquake theory.—This also may be dismissed briefly. It could not account for the period nor for the fact that, in such a comparatively small area as the Gulf of St. Lawrence or the Bay of Fundy, some gauges always show secondary undulations, others never show any.

(3). "Seiche" theory.—Four years ago I advanced the suggestion* that the undulations were of the same nature as the "seiche" movements observed by Forel and others on the Swiss lakes, that is undulations of a partially confined body of water under the force of gravity, like the "wish-wash" of water in a bowl when disturbed. This explanation seemed to fit entirely the oscillations at St. John (both harbor and river), calculations from the soundings in the vicinity showing a remarkable agreement of calculated and observed period on the supposition that the oscillations were not of the fundamental or gravest mode (uninodal), but of the first higher mode (binodal). As will be seen later, it does not accord so well with other cases.

It is to be noted that the above theory is chiefly concerned with the *period* of undulation, leaving the initial disturbance of equilibrium to be accounted for by atmospheric conditions or by neighbouring oceanic disturbances invading the basin of oscillation.

(4). "Atmospheric billows" theory.—Mr. F. Napier Denison has advanced the interesting and ingenious theory† that "the secondary undulations are due to atmospheric waves or billows set up in the upper atmosphere" at the boundary between "the upper part of the lower stratum of air and the higher stratum which is less dense and rapidly moving polewards." To account for "how atmospheric waves, which correspond to only a few hundredths or thousandths of the barometric inch at the earth's surface, cause such rapid and extreme oscillations" of the water surface, Mr. Denison adds that "the atmospheric waves or billows, in passing over the

* This Journal, iii, 1897, where a full account of the bibliography of seiches will be found. In the paper here referred to, following the analogy of sound waves I used the word node (in binodal, trinodal, etc.), as meaning a place of minimum longitudinal motion. Finding it customary to use node in reference to vertical movement when speaking of water waves, I have changed the notation in the present paper.

† Articles already cited.

surface of the sea, tend to form minute undulations upon the surface, corresponding to the length of the billows, which, as they move farther into the bay, become magnified as they reach narrower and shallower portions." In a later paper Mr. Denison repeats the above views and states that "there is a marked relative correspondence in amplitude between the barometric and water undulations," adding many interesting speculations concerning a possible connection between secondary undulations and warm and cold waves, precipitation, etc., but we are not concerned with them at present. (It may be added that Mr. Denison's papers are very brief, containing but six pages in all and, excepting one case of secondary undulation at Halifax, giving no references in detail to the coincidences of barometric, meteorological and tidal records on which the theory is founded, so that, in again traversing the same ground, I was unable to refer directly to the evidence for the deductions.)

The above is primarily a theory of the *origin* of the disturbances. It would at first sight not seem incompatible with any other explanation of the period.

These theories will be tested in the sections that follow.

Comparison of observed and calculated periods.

In order to test how far the seiche theory, advanced for the cases at St. John, would apply in general, I calculated from the admiralty charts the width and mean depth of basins and channels immediately adjacent to several of the above mentioned stations and therefrom deduced the period in which such bodies of water should vibrate.

Station.	Basin or Channel.	Period observed.	Period calculated. binodal.	uninodal.
St. John Harbor	Bay of Fundy	41 m.	43·5 m.	87 m.
St. John River	(small bay)	35 sec.	37·5 s.	72 s.
Westport	St. Margaret's Bay	16 m.	14 m.	28 m.
Yarmouth	N.S. to Lurcher Shoal	30 m., 69 m.	30 m.	60 m.
Souris	Northumberland Strait	21 m.	45 m.	90 m.
Farther Point	St. Lawrence River	21 m.	50 m.	100 m.
S.W. Point	Anticosti to Gaspé	19 m.	30 m.	60 m.
Forteau Bay	Strait of Belle Isle	16 m.	23 m.	47 m.

The above calculated periods can only be considered as roughly approximate. They suffice, however, to show that, except in the first four cases, the periods cannot be those of binodal seiches, and if they are seiche periods at all the seiches must be multinodal (e. g., at S.W. Pt. and Forteau Bay trinodal, at Farther Pt. quinque nodal). Local peculiarities, for example variation in the depth of the basin, would,

of course, tend to set up partial vibrations (as would inequalities in the thickness of a cord.)

The above comparison then, while not at variance with the seiche theory, affords only partial confirmation of it. To other cases no calculation can be applied. Digby is on a basin of irregular form which may have somewhat regular vibrations, though theory is at present inadequate to calculate the period of such irregular bodies of water. The same is true of Halifax, with the peculiarity that the basin is one marked out by a series of banks (Roseway, Le Have, Emerald, &c.) which, while not coming to the surface, bound an irregular bay, attaining at places a depth of 300 fathoms. Pictou and Quaco are also on partially enclosed bays (reckoning the Quaco "ledges" as part of the boundary of the latter). At Grindstone Island (in the middle of the Gulf) and St. Peter's Bay (on the outer or Gulf side of P. E. I.) the undulations are always or nearly always irregular, as might be expected of points near the middle of such an irregular body of water as the Gulf of St. Lawrence.

The case of St. Paul's Island (at which there are periods of 4·7 and 12·9 m.) might be thought at first inexplicable, as the island is in Cabot strait between prominent headlands on either side, with no suggestion on an ordinary map of a channel or basin of oscillation. But a glance at the nautical chart (of 1891) shows that, while the main channel has a great depth (approaching 300 fathoms), on both sides of this channel, there is an abrupt change of depth and gradient and the contours of, say, 100 fathoms on opposite sides of this channel run roughly parallel for a distance greater than the width of the strait. Moreover, on the Cape Breton side and adjacent to St. Paul's Island, there is a depressed basin, only part of the boundary of which reaches the surface. Thus we have clearly the conditions for secondary undulations according to the "seiche" theory.

The case of Yarmouth presents a striking peculiarity. The seiche theory gives a very satisfactory account of the periods at Yarmouth; but why should the period at high tide be regularly twice as great as at low tide? The mere change in depth of the channel can have no such striking effect. The only other factor that changes with the state ot the tide is the direction of the *tidal current.* I find myself unable to explain satisfactorily why the reversal of the current should have such a marked effect. A thorough examination and elucidation of this peculiar case might lead to a more satisfactory explanation of the whole subject than has yet been offered. It might also be noted that at St. Paul's Island the unusual elevation of water level at both high and low tide, on the

occasions on which the period shifts from 4·7 m to 12·9 m, would also mean some marked change in the currents through Cabot Strait.

Does the varying depth of the Bay of Fundy affect the period at St. John?

As stated in an earlier paper (already referred to), a conclusive test of the seiche theory would be the following. The period of oscillation of a body of water varies inversely as the square root of the mean depth. Now the mean depth of the Bay of Fundy at low water is something over 200 ft., and at high water (on the average) nearly 20 ft. greater. The width of the bay at St. John being about 40 miles, it can be calculated that the period of secondary undulation at St. John should, according to the seiche theory, be about 1·8 m. greater at low water than at high water.

To test this I selected 106 tracings of particularly well-marked secondary undulations at St. John. These I measured for period with the greatest care. Then I sorted them out into high tide undulations and low tide undulations, finding 49 of the former and 57 of the latter. The mean period for high tide was 40·5 m. and for low tide 41·7—a difference of 1·2 m. Now this was wholly confirmatory of the seiche theory. That the difference 1·2 should come out less than the calculated difference 1·8 is readily understood from the fact that the undulations nearly always continued before and after high and low tide, so that the effective difference of depth would be less than the mean range of the tide.

To test the probability of the above differences being mere chance, I calculated "the probable error" of each series, and found it to be the same—0·4 m. Now the mean of each series only differs from the mean of the whole by 0·6 m. From this it is a comparatively simple problem to calculate what is the probability that by mere chance the mean period at high tide was 0·6 m. higher than its real value *and* the mean value for low tide the same amount lower. The probability is $\frac{1}{18}$ or the chances are 12 to 1 against the difference being due to the mere chance distribution of somewhat rough determinations of what is really the same quantity.

Thus this test, while very strongly in favor of the seiche theory, is not conclusive.

Connections between undulations at different places.

Taking first points on the Atlantic and the adjacent Bay of Fundy, it is shown by a comparison of the records at Halifax, Yarmouth and St. John, that marked oscillations at one are

usually accompanied by marked oscillations at the other. Conspicuous examples are, Jan. 13th, 1898; Nov. 26 and 27, 1898; Dec. 23, 1898; Dec. 31, 1898; Jan. 1 and 2, 1899, &c. The exceptionally strong undulations on these dates coincided with severe storms from the south on the Atlantic. But this connection is not quite invariable, e. g., on Aug. 14, 1898, strong oscillations of 15 inches at Yarmouth were not accompanied by any exceptional oscillations at St. John or Halifax.

Turning now to a comparison of stations on the Atlantic and stations on the Gulf of St. Lawrence, it is found that in general no such connection exists. For instance, in my note book, opposite each of the cases of marked oscillations at St. John, Halifax, and Yarmouth, just enumerated, I have a note of the state of affairs at St. Paul's Island, South West-Point and Forteau Bay, and the notes, with two exceptions, are "no oscillations," "particularly slight," "less than normal," "only normal." In the two exceptional cases the note runs "above normal," and these two notes refer to St. Paul's Island, which, being in Cabot Strait, belongs as much to the Atlantic as to the Gulf.

The lack of connection here noted seems a point of such importance that I will illustrate it by another example. Pictou on the Gulf and Halifax on the Atlantic are on opposite sides of Nova Scotia and only about 100 miles apart, a much shorter distance than separates St. John and Halifax. The large tide gauge at Pictou was only maintained during the summer of 1896. I have compared the records from it with those from Halifax. In the following from my note-book the first note refers to Pictou, the second to Halifax:—July 3, "marked"—"normal"; July 17, "nothing"—"strong"; July 25, "nothing"—"strong"; Sept. 19, "nothing whatever"—"strong"; Sept. 20, "feeble"—"strong"; Sept. 24, "weak" "strong"; Sept. 28, "strong"—"normal"; Nov. 1, "feeble"—"strong"; Nov. 11, "nothing"—"strong"; Nov. 14, "nothing"—"strong." These were, I believe, the only cases in which the undulations at either place were decidedly above normal and therefore suitable for founding such a comparison on. Thus there is no correspondence between the occurrences of secondary undulation at Halifax and Pictou.

Now, according to the theory of atmospheric billows, two points so comparatively close together as Halifax and Pictou should show a close correspondence of times of strong undulations; for it cannot be conceived that "atmospheric waves of from 15 to 30 kilometers long" whose "average velocity is 60 miles per hour in summer"* could raise strong secondary

* Denison, Proc. Can. Inst, Jan. 16, 1897, p. 30; Feb. 6, 1897, p. 58.

undulations at Halifax and fail to affect Pictou and *vice versa*. This discrepancy is still more marked when it is considered that Pictou is almost exactly N.E. from Halifax, and it is on the poleward current, moving N.E., that Mr. Denison relies for the production of the atmospheric billows in question. (It is to be understood here that I am not discussing the existence of atmospheric billows but the effects attributed to them.)

On the other hand, the close correspondence between Atlantic (including Bay of Fundy) ports, in contrast with the lack of correspondence between Atlantic and Gulf ports, is strongly in favor of the view that the disturbance at any port is usually due to the transmission by water of distant disturbances of the general body of water caused by storms.

Secondary Undulations and barometric records.

Any connection between atmospheric disturbance and secondary undulation should, of course, be shown by a comparison of tidal and barometric records. I have carefully compared the tracings of secondary undulation at St. John for the summer of 1896 with tracings of the St. John barograms for the same date. These latter I owe to the kindness of Mr. Bell Dawson. They were made as carefully as possible with a hard, sharp-pointed pencil. The dates are June 8, 9, 10, 21, 22, 23; July 8, 25; Sept. 1, 5, 6, 7, 18, 19, 20, 22, 23—the only occasions, I believe, on which marked secondary undulations occurred at St. John during the summer of 1896. A careful comparison of both series of records shows that (1) disturbances of atmospheric pressure usually occur about the times of secondary undulations, and, in general, the stronger the atmospheric disturbances the stronger the water undulations, but (2) the atmospheric disturbances rarely if ever begin and end at the same time as the water undulations; they usually precede but sometimes seem to follow the latter, and the secondary undulations may continue for several hours (e. g. 12 hours on Sept. 20) after the barometric disturbances have ceased; (3) in no case is there any suggestion of regular periodicity in the barometric disturbances such as there is in the water undulations.

The utmost therefore that it seems to me possible to deduce is that secondary undulations usually accompany storms near by or at a distance, but that the *period* of the oscillations at any place is wholly unaccounted for by atmospheric conditions. (This is also in accord with an extensive series of comparisons that I made, of the Atlantic Pilot Charts and the Canadian Weather Maps on the one hand and the records of secondary undulations on the other—into the details of which it seems superfluous to enter.)

Mr. Denison, however, states as the first of the ten "chief points deduced" by him (but without any indication of the details of the evidence), "that the undulations are due to the direct action of atmospheric waves upon the surface of the water *at stations* [italics mine] and not to ground swells, due to distant storms, or 'seiche' movements, as found upon lakes during atmospheric disturbances." Now with the former or positive part of this conclusion the evidence I have adduced above does not seem at all compatible; the latter or negative part, denying any other agency, can only mean the sufficiency of the theory of atmospheric waves to account for the *period* as well as the *origin* of the secondary undulations, which again is wholly at variance with the fact that the barometric disturbances show no periodicity and the water indulations continue regular and periodic after the barometric record has become quite smooth. The second point enumerated by Mr. Denison is that "there is a marked relative correspondence in amplitude between the barometric and water undulations." This I also have found to be the case, provided it be not understood to imply that the two kinds of undulations always (or even usually) occur or continue at the same time. (I would also take some exception to the word "marked.")

From the above statements in Mr. Denison's paper it has been inferred by some readers that he found regular atmospheric oscillations coinciding with and accounting for the water undulations both as regards origin and period. Mr. Denison does not directly say so and this cannot be his meaning. There is absolutely no evidence for it in the cases I have referred to above, covering a whole summer of St. John records. How in fact is it conceivable that atmospheric billows many miles in length and moving with a velocity of from 60 to 112 miles per hour, should always have a periodicity of half a minute at the mouth of the St. John River—41 m. at St. John Harbour (less than a mile away)—12½ m. at Quaco (20 miles away)—69 m. at Yarmouth at high tide, but 30 m. at low tide—20 m. at Halifax on some occasions, but 33 m. on other occasions and occasionally probably both—47 m. at St. Paul's Island—29 m. at Pictou, etc.?

Effect of Contraction of Bay.

Reference must be made to another point in the "atmospheric billows" theory. To account for such slight variations of pressure causing such large variations of water level, it is necessary to suppose that, as in the case of ordinary tides, "minute undulations as they move farther into the bay become magnified as they reach narrower and shallower portions."

Yet, if we consider the Bay of Fundy, exactly the opposite is true. Yarmouth is at the mouth of the bay and there the oscillations sometimes reach 5 ft.; St. John is half way up the bay and there the oscillations never exceed 2 ft.; while at the head of the bay, where, as is well known, the rise of the tide is enormous, Hopewell Cape, Parrsboro and Windsor show no secondary undulations. Again Carleton, Q., is at the head of Chaleurs Bay, but the undulations there are particularly slight. At Cape Tormentine, which is in the narrowest part of Northumberland Strait, there are no secondary undulations and the same is true of Charlottetown, while they are very frequent at Pictou and Souris near the entrance of the Strait.

On the other hand, it is to be noted that, according to the seiche theory, secondary undulations would not be expected in shallow basins such as those at the head of the Bay of Fundy; since, if oscillations of the whole body of water were started, they would rapidly die away in such shallow water.

On the whole, the theory of "atmospheric billows," while ingenious and attractive, seems to me wholly untenable as regards the *period* of undulation and very improbable as regards the *origin* of such undulations (I am not of course referring to the agency of atmospheric disturbances in general), whereas the "seiche" theory offers the only tenable explanation hitherto advanced, but leaves points still unexplained.

A general view of the vibrations of large bodies of water.

It may be well at this stage to make explicit a general view implied in the preceding discussion. It seems probable that a large body of water, such as the Gulf of St. Lawrence or the Bay of Fundy, when disturbed breaks up into smaller areas of oscillation, determined by irregularities in the level of the bed; these minor basins oscillate to a considerable extent independently, at rates determined by their respective dimensions and the depth to which they are filled, the oscillations being at times in the fundamental mode, but at other times in some higher mode, but with a tendency to the establishment of some particular mode that gives a characteristic period to the basin. Frequently, owing to the co-existence of different modes of vibration, and owing to invasions from the vibrations of neighboring basins, the vibrations of each minor basin become more or less irregular; but such irregularities are comparatively rare in a part of the main basin with only slight irregularities of bottom and bounded on opposite sides by nearly parallel coasts (as is the case of the Bay of Fundy at St. John, where the records show a more constant period of undulation than at any other tidal station on the eastern coast of Canada).

Another possible source of periodic fluctuations under certain circumstances.

Can any other explanation be found for the definite period that characterizes the regular undulations at each place? I can think of but one other.

Most people who have, from the shore, watched the progress of a ship have noticed the regular series of waves thrown off from the bow. If the ship be at anchor in a rapid current, a similar series of waves are produced, reaching any point on the shore at equal intervals. In general it may be stated that "a line of pressure athwart a stream flowing with velocity c produces a disturbance consisting of a train of waves of length $2\pi c^2/g$ lying on the down-stream side."* The effect of inequalities in the bed would be similar. Would an island, shoal, bank or headland, past which a current is setting, cause a similar series of waves of a length and period commensurate with the size of the obstruction? For example, St. Paul's Island divides two opposite currents between the Atlantic and the Gulf of St. Lawrence; at Yarmouth the direction of the tidal current is reversed with the tide and the Lurcher shoal is a prominent obstruction.

I do not attempt to consider this suggestion in detail. It seems to me a much less likely explanation than the theory of stationary undulations or seiches; but no adequate attempt, in a brief space, to test its applicability would be possible, even if data as regards currents, etc., were to be had.†

Summary of preceding.

A. Facts.

1. Secondary undulations occur at most of the places where observations have been made, the undulations at any place being sometimes irregular but at other times of a regular periodic nature.

2. When the undulations are regular, the period of undulation has a distinct and characteristic value for each place, varying, from place to place, from less than a minute to over an hour.

3. At some places at least two systems of regular periodic undulations of different periods are found; they usually occur

* Lamb's Hydrodynamics, § 228, where illustrations of such waves from experimental papers by W. and R. E. Froude and theoretical papers by Lord Kelvin and Lord Rayleigh are given.

† Moreover, in making this somewhat vague suggestion, I am aware that the formula quoted above for the length of such waves is independent of the size of the obstruction: but this does not preclude the possibility that hydrodynamical theory, applied to the case of very large obstructions, might be able to account for all the phenomena.

at different times, sometimes they occur together; at one station the period at high tide is entirely distinct from that at low tide.

4. Undulations usually occur simultaneously at stations on the Atlantic Ocean (including the Bay of Fundy), but there is apparently no connection between this occurrence at stations on the Atlantic and stations on the Gulf of St. Lawrence.

5. Barometric records show no similar periodic oscillations, but they do show disturbances at or about the times of marked secondary undulations.

6. As the head of a bay is approached secondary undulations do not become more marked and frequent, but apparently less.

7. According to the best available evidence the period at St. John is less the greater the depth to which the tide fills the Bay of Fundy.

B. Deductions.

1. The theory of atmospheric billows does not and cannot explain the characteristic local *period* of secondary undulations.

2. As an explanation of the *origin* of secondary undulations the theory of atmospheric billows seems at variance with certain facts of fundamental importance to the theory (see 4 and 6 under A, above).

3. The seiche theory, which is fully established in the case of lakes, has a high inherent probability and is not at variance with any ascertained facts and in certain respects receives strong confirmation (see 2, first part of 3, 4, 6, and 7, under A, above).

4. Even if the "seiche" theory be correct, it still remains to be explained why the oscillations at certain places are usually binodal, at others trinodal, etc., and why in one striking case (Yarmouth) the oscillations are always uninodal at high tide and binodal at low tide.

5. The whole subject is apparently much more complex than has hitherto been supposed and it may yet be found that no single explanation will apply to all cases.

Art. XIV.—*Mathematical Notes to Rival Theories of Cosmogony;* by O. Fisher.

The following mathematical notes are supplementary to the discussion of "Rival Theories of Cosmogony" in the June number of this Journal (pp. 414–422).*

Let—

r = distance of any point from the earth's center.
a = the earth's mean radius in feet.
$\log a$ = 7·3201961.
qa = 2·4605 in radians = 140° 58′ 35″.
s = the density of the surface rock (2·75).
gs = the weight of a cubic foot of the same, viz: 171·875 pounds.
ρ = the density at any depth.
p = the pressure in pounds per square foot at the depth where the density is ρ.
c = the coefficient of compressibility at that depth measured by atmospheres.
J = Joule's equivalent, 772.
σ = specific heat of rock.

(1) Laplace's law of density is, $\rho = Q\frac{\sin qr}{r}$,

where Qq is the density at the center, 10·74,
and qa = 2·4605 in radians = 140° 58′ 35″.

(2) The relation upon which the following calculations depend is one which I have proved in the Appendix to my "Physics of the Earth's Crust" (p. 32), viz:

$$p = \frac{gs}{2s^2}\frac{a}{1 - qa\cot qa}(\rho^2 - s^2);$$

$$= B(\rho^2 - s^2), \text{ suppose,}$$

which, reduced to numbers, gives for the relation between the pressure measured in pounds per square foot and the density,

$$p = 5{\cdot}8855 \times 10^7\{\rho^2 - (2{\cdot}75)^2\}.$$

Hence we see what is the value of the constant B, which is implied in Laplace's law of density, viz: 5·8855 × 10⁷ in the above units.

Also since $cp = \frac{\rho - s}{\rho}$ or $c = \frac{1}{p}\frac{\rho - s}{\rho}$,

* Attention is called to the following *erratum:* On p. 421, for 20 miles read 40 miles.

we get for the compressibility in pounds per square foot,

$$1{\cdot}699 \times 10^{-8} \frac{\rho+s}{\rho},$$

which reduced to atmospheric pressure is

$$c = 3{\cdot}6069 \times 10^{-8} \frac{\rho+s}{\rho}.$$

(3) The above relation between the pressure and the density renders it easy to calculate the work of compression, and thence the temperature;

for $$work = -\int_{v_0}^{v} p\,dv,$$

where $$p = B(\rho^2 - s^2).$$

And since $$\frac{v}{v_0} = \frac{s}{\rho},$$

$$\therefore\ dv = -v_0 s \frac{d\rho}{\rho^2};$$

and $$work = v_0 s \times 5{\cdot}8855 \times 10^7 \int_s^\rho (\rho^2 - s^2) \frac{d\rho}{\rho^2}$$

$$= v_0 s \times 5{\cdot}8855 \times 10^7 \left(\rho + \frac{s^2}{\rho} - 2s\right)$$

$$= v_0 s \times 5{\cdot}8855 \times 10^7 \frac{(\rho - s)^2}{\rho}.$$

And the *temperature* $= \dfrac{work}{J \times specific\ heat \times mass}$

Mallet's value for the specific heat of rock is 0·199. The mass will be $s \times v_0$, which will divide out, and thus

$$temperature\ Fah. = \frac{B}{J\sigma} \frac{(\rho - s)^2}{\rho}.*$$

This will give the temperature so long as we can reckon upon the specific heat remaining constant, which probably will not be the case for the higher temperatures, and certainly not so if the substance becomes liquid or gaseous, when the latent heats will disappear, but it will be sufficient to show what enormous temperatures such condensation would produce, even if the specific heat should be as great as that of water.

* The log of the coefficient $\frac{B}{J\sigma}$ is 5·5833156.

(4) The lava issued from Etna at 1932° Fah. We can find the depth at which that temperature would have been produced by compression of surface rock.

We have $$\textit{temperature Fah.} = \frac{B}{J\sigma}\frac{(\rho - s)^2}{\rho}.$$

Applying this to the present case, we get

$$\frac{(\rho - s)^2}{\rho} = 5{\cdot}043 \times 10^{-4} = 2m, \text{ suppose,}$$

whence $$\rho - s = m \pm \sqrt{(s+m)^2 - s^2}.$$

Since the surd is greater than m and $(\rho - s)$ must be positive, we must use the upper sign; we then obtain

$$\rho - s = 0{\cdot}120.$$

This will be the increase of density to the depth where the temperature is 1932° Fah. or $\rho = 2{\cdot}87$.

We then require the value of r which will make

$$Q\frac{\sin qr}{r} = 2{\cdot}87.$$

It appears on trial that a depth $a - r$ of about 40 miles satisfies this condition. It follows that the depth at which the temperature 1932° Fah. would be produced would be about 40 miles.

TABLE.

Tenths of radius.	Approximate depth in miles.	Densities by Laplace's Law.	Pressure in atmospheres.	Condensation $\frac{v_0 - v}{v_0}$ requisite to reduce surface rock to density.
1·0	0	2·75	0·000	0·000
0·9	400	3 88	$2{\cdot}181 \times 10^7$	0·29
0·8	800	5·03	$5{\cdot}055 \times 10^7$	0·45
0·7	1200	6·17	$8{\cdot}460 \times 10^7$	0·55
0·6	1600	7·25	$12{\cdot}344 \times 10^7$	0 62
0·5	2000	8·23	$16{\cdot}867 \times 10^7$	0·67
0·4	2400	9·09	$20{\cdot}698 \times 10^7$	0·70
0·3	2800	9·80	$24{\cdot}370 \times 10^7$	0·72
0·2	3200	10·32	$27{\cdot}030 \times 10^7$	0·73
0·1	3600	10·64	$29{\cdot}052 \times 10^7$	0·741
0·0	4000	10·74	$30{\cdot}061 \times 10^7$	0·744

Art. XV. — *Studies of Eocene Mammalia in the Marsh Collection, Peabody Museum;* by J. L. Wortman.

[Continued from vol. xi, p. 450.]

Family Viverravidæ.

Miacidæ Cope (in part), Tertiary Vertebrata, 1884; *Miacidæ* Scott, Jour. Acad. Nat. Sci., Phila., 1886, vol. ix, p. 169; *Viverravidæ* Wortman and Matthew, Bull. Amer. Mus. Nat. Hist., 1899, p. 136.

A family of small or medium-sized Carnassidents ancestral to the viverrines, ranging in time, as far as at present known, from the Torrejon to the Bridger Eocene, having pentedactyle limbs with ununited scaphoid, lunar, and centrale of the carpus, a civet-like perforation of the transverse process of the atlas for the passage of the vertebral artery, a well-developed anterior basal cusp upon the superior sectorial, no antero-posterior femoral curvature and no postero-internal cingular cusp upon the superior molars. Blades of the superior sectorial separated by a deep slit-like notch, and molar formula either $\frac{3}{3}$ as in the primitive Canidæ, or $\frac{2}{2}$ as in the Viverridæ.

In defining the foregoing family, it is necessary to distinguish it from the contemporary Canidæ on the one hand and its successors, the Viverridæ, on the other. As regards the former, while in all probability they have been derived from a common source, yet they have departed sufficiently in the direction of their subsequent and final development to be recognizable. There are two characters that appear to be entirely distinctive, one of which relates to the atlas and the other to the presence or absence of an anterior basal cusp on the superior sectorial; associated with these is the lack of curvature of the femur and no disposition whatever towards the formation of a postero-internal cingular cusp on the superior molars,—features which have characterized all lines of Canidæ in some stage of their development. From the Viverridæ, they can be distinguished not by any essential characters of fundamental importance, but only by the possession of such primitive and archaic features as we should reasonably expect to find in the ancestors of the modern viverrines. The more important of these relate to the separate condition of the scaphoid, lunar, and centrale of the carpus; the presence of a third trochanter on the femur; the large size and internal position of the lesser trochanter; the slight grooving of the astragalus, and the large size of the deltoid crest of the humerus. To this may be added the greatly inflated and modernized condition of the otic bulla in the living Viverridæ.

From the Mustelidæ, as far as at present known, the Viver-

ravidæ are distinguished not only by the possession of the primitive characters above enumerated, but by two very trenchant dental peculiarities which the mustelines exhibit, viz.: the great antero-posterior enlargement of the internal portions of the crowns of the superior molars, and the absence of the deep slit-like notch separating the two blades of the superior sectorial. These, together with the absence of the anterior basal cusp on the superior sectorial, the great breadth of the base of the skull, the small rugged bulla, and long bony spout-like meatus, constitute the real distinguishing features by which the Mustelidæ may be separated from the Viverridæ. The skeletal characters are otherwise much alike in the two families, the manner of perforation of the transverse process of the atlas being variable in the mustelines.

With reference to the Felidæ, it may be said that our knowledge is not so perfect regarding their origin and Eocene representatives as it is with respect to other living families, so that it is well nigh impossible to make a satisfactory comparison. If the Palæonictidæ are the forerunners of the felines, which seems so exceedingly probable, then the two groups belong to different phyla and may be distinguished by the structure of the superior molars. In the Palæonictidæ, it is the posterior part of the crown of the first superior molar which is elongate; whereas in the Viverravidæ, it is always the anterior part of the crown which is the longer.

Viverravus Marsh.

Viverra, a civet cat; and *avus*, a grandfather. This Journal, August, 1872; *Didymictis* Cope, Tertiary Vertebrata, 1884; *Viverravus* Wortman and Matthew, Bull. Amer. Mus. Nat. Hist., 1899, p. 136.

A group of small or medium-sized civet-like Carnivores, with a number of species distributed throughout the Eocene from the Torrejon to the Bridger inclusive. They are characterized by having the dental formula I. $\frac{3}{3}$, C. $\frac{1}{1}$, Pm. $\frac{4}{4}$, M. $\frac{2}{3}$, of which the two superior molars of the dentition have broad tubercular crowns, with great transverse extension of the anterior border, the two external cusps being unequal and placed well inwards from the external margin of the crown; the second inferior molar, tubercular or becoming so, and much smaller than the first; the inferior sectorial having a high trigon, with oblique principal shear, a well-developed posterior shear, and a relatively large, more or less basin-shaped heel; and premolars having posterior accessory cusps.

In his original description of this genus, Professor Marsh says: "A much smaller Carnivore, about the size of the mink, is represented in our collections by two lower jaws with teeth, and a sectorial upper molar of one individual and portions

apparently of several others. The lower jaws in this genus are long, slender and compressed; the last two lower molars are tubercular. Both have the posterior part of the crown quite low and the anterior half elevated and composed of three angular cusps. The four teeth anterior to these are much compressed. The upper flesh tooth closely resembles that in some of the Viverridæ and the genus should probably be referred to that group."* The species thus far referable to this genus are numerous, of which one, *V. haydenianus*, from the Torrejon; four, *V. leptomylus*, *protenus*, *massetericus*, and *curtidens*, from the Wasatch; and two, *V. gracilis* (*dawkinsianus*) and *altidens*, from the Wind River beds, have been described by Cope. It will thus be seen that the genus has a very great vertical range, greater, in fact, than any known contemporary group of mammals throughout the whole Eocene. With the close of the Bridger epoch, according to our present knowledge, the genus disappeared from this country, since no remains of it have as yet been found either in the Uinta, White River, or John Day deposits.† It is possible, however, that the group may have continued to exist to a much later date on this continent, and that they retreated to the southward along with the tropical fauna which disappeared from Wyoming at the close of the Eocene. It is possible, therefore, that their remains will yet be found in the Miocene of the South, but this, of course, is merely conjectural.

Viverravus gracilis Marsh.

Didymictis dawkinsianus Cope, Tertiary Vertebrata, 1884, p. 310.

The type, figures 18, 19, consists, as Professor Marsh has stated, of parts of both mandibular rami and a superior sectorial, but there are at least twenty individuals of the species represented in the collection by various fragments. Of the type the right ramus is the more perfect of the two, and carries the third and fourth premolars and the first and second molars. The alveoli for the first and second premolars, together with that for the root of the canine, are represented. All the premolars are two-rooted, even the first, which among the Carnivora is very generally a single-rooted tooth. The third and fourth premolars have high pointed crowns, with anterior and posterior basal cusps, together with a distinct and trenchant accessory cusp. The sectorial has the trigon much elevated

* Loc. cit, p. 7, of separata.

† A possible exception to this statement may be found in the imperfectly known genus *Bunælurus* of Cope, from the White River Oligocene of Colorado. When more fully known, it will not be at all surprising to find that this genus is a direct descendant of *Viverravus*.

and of greater antero-posterior length than the heel; the anterior and internal cusps are equal in height, but the external was evidently much higher; it is somewhat damaged and does not display its full length. The principal shear is oblique, and there is a posterior shear which bites against the anterior edge of the first molar. The heel is relatively short, wide, and basin-shaped; the external part of the basin is the thicker and more elevated. The second molar is much smaller than the first, but displays practically the same structure; the trigon, however, is much less elevated, and the shears are imperfect. The external and internal cusps are of equal size and height, but the anterior is much smaller and lower. The heel is relatively longer than in the first molar, being obtusely pointed behind, where it is terminated by a low but distinct cusp; from this cusp a low ridge is continued forwards and inwards to enclose the inner

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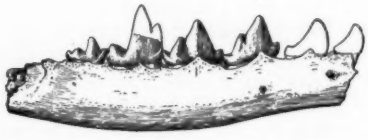

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FIGURE 18.—Right lower jaw of *Viverravus gracilis* Marsh; outside view; natural size. (Type.)

FIGURE 19.—Superior sectorial of same; outside and inside views; natural size. (Type.)

FIGURE 20.—Superior sectorial of *Viverricula* sp.; natural size.

part of the basin. Externally there is a stronger cusp situated in advance and to the outer side of the last mentioned cusp, which furnishes the external boundary of the basin.

The superior sectorial displays the typical laniary structure of the more highly-developed Carnassidents; it is composed of a pair of external cusps, laterally flattened and elongated in such a manner as to form a pair of very effective shearing blades. Of these, the anterior is the larger and more elevated, being separated from the posterior by a deep vertical notch, which appears as a narrow slit upon the lingual surface. At the antero-external angle of the larger cusp is seen a lower, but very distinct, anterior basal tubercle. Internal and opposite to the anterior edge of the main external blade is placed a relatively large, pointed internal cusp of about the same proportions of that of the Genet. In fact, the whole dentition, as far as known, is strikingly like that of this living species.

In his description of *V. dawkinsianus*, Professor Cope states that the first premolar of the inferior series is a single-rooted tooth. If this is true (which I am inclined to doubt), the two

species are distinct and we have the anomalous condition of the latest known species of the genus being less specialized in this particular than its immediate predecessor. The species is distinguished from all the others by its small size and the short broad heel of the lower sectorial. The principal measurements of the type are as follows:

Length of inferior molars and premolars	28· mm
Length of inferior molars	9·5
Length of sectorial	5·2
Transverse diameter of sectorial	3·4
Antero-posterior diameter of superior sectorial,	7·
Transverse diameter of superior sectorial	3·5

The type specimen was found at Grizzly Buttes, by G. G. Lobdell; other specimens were obtained at various levels from the upper to the lower part of the horizon.

Viverravus minutus sp. nov.

This species is represented in the collection by remains of at least six individuals, of which the lower jaws alone, in varying degrees of completeness, are preserved. That which is here

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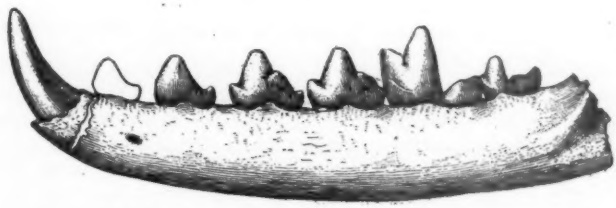

FIGURE 21.—Left lower jaw of *Viverravus minutus* Wortman; outside view; three times natural size. (Type.)

selected as the principal type, figure 21, is a left ramus carrying the molars and the three posterior premolars in good preservation. The specimen in question is broken just posterior to the base of the coronoid and also in front in the region of the symphysis, but it includes the base of the canine alveolus as well as that for the first premolar. The crowns of the molars are somewhat worn, so as not to display very clearly the character of the cusps of the heels. On this account, I associate with it another specimen as a cotype, also a left mandibular ramus of the left side, in which the heels of these teeth are more perfectly preserved. A third fragment of a jaw carries the canine in perfect condition.

The character of the species, as exhibited by these three specimens, may be stated as follows: It is considerably smaller than *V. gracilis;* the first premolar is two-rooted; the third and fourth premolars have anterior and posterior basal cusps and posterior accessory cusps which are absent on the second; the heel of the sectorial is proportionally narrower, and not quite so distinctly basin-shaped, as in *V. gracilis.* The last molar has a distinctly narrower, more trenchant heel and does not exhibit the two cusps seen in *V. gracilis.* The canine is long, slender, and considerably recurved. The remains indicate an animal of the size of the common weasel, the slender jaws and sharp cutting teeth giving evidence of a very highly carnivorous habit.

Length of inferior molars and premolars	21· mm
Length of inferior molars	7·
Length of first molar	4·5
Width of first molar	2·5
Depth of jaw at interior sectorial	4·5

The principal type specimen was found on Dry Creek, Bridger Basin, by Messrs. Lamothe and Chew, although other specimens are from Grizzly Buttes.

Oödectes herpestoides gen. et sp. nov.

Oön, an egg; and *dectes*, a biter, in allusion to the habits of the Ichneumon.

A genus of small viverrine-like animals, having the dental formula I. $\frac{3}{3}$, C. $\frac{1}{1}$, Pm. $\frac{4}{4}$, M. $\frac{3}{3}$, with trigon of inferior molars high, the cusps sub-equal, and the principal inferior shear high and very oblique; inferior molars with trenchant or slightly basin-shaped heels; transversely extended superior molars, with tubercular crowns; anterior and posterior external angles of first molar about equally extended, and antero-external angle of second more extended than posterior; a superior sectorial, with sharp blade-like principal cusps, and premolars without posterior accessory cusps.

The remains upon which this genus and species are founded consist of the larger part of the skeleton of one individual, considerably broken, but at the same time with nearly all the characteristic portions represented. I associate with this type two other individuals, one of which is represented by a left lower jaw, broken away at the base of the coronoid, but containing all the teeth, with the exception of the incisors, in a state of good preservation; the third specimen is the posterior portion of a left mandibular ramus of an immature individual, in which the fourth premolar was just coming into position.

Dentition.—With the exception of a single isolated tooth which apparently belongs to the upper series, the incisors are not preserved in any of the three specimens, but the bases of the alveoli for those of the lower jaw can be indistinctly made out. They were three in number and arranged apparently as in the Paradoxures, without having the second one pushed back out of line, as is frequently found in the Canidæ and some Viverridæ. The canine, figure 22, is relatively large, high, pointed, and recurved; there is a faint vertical external groove, and a broader shallow one upon the internal face of the crown; on the posterior and external surface is an extensive worn area, where the tooth impinged upon its fellow of the upper series.

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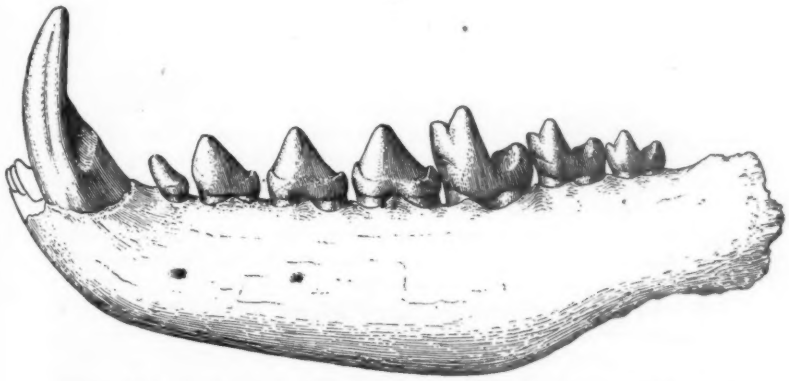

FIGURE 22.—Left lower jaw of *Oödectes herpestoides* Wortman; outside view; three times natural size. (Cotype.)

The premolars are four in number, with rather short, stout, thick crowns, having more or less of a tendency towards the development of strong internal cingula and cusps as seen both in the Paradoxures and Herpestinæ. The first is small, single-rooted, with an obtusely-pointed crown directed well forwards like the corresponding tooth in *Herpestes griseus*, and separated from the canine by a short interval. The second is abruptly larger, two-rooted, and has a small, indistinct posterior basal cusp. The third and fourth are still larger; their crowns are relatively thick, with obtuse points and inconspicuous anterior and posterior basal cusps; there are no posterior accessory cusps present, except a slight indication of one in the last premolar, as in *Herpestes*, to which the series of teeth, including the canine, bears a very striking resemblance.

The molars show many peculiarities for a Carnivore, and at first glance might indeed be readily mistaken for an Insecti-

vore; but a careful examination clearly reveals their carnivorous character. As is usual among the Carnassidents, the first molar is the largest of the series, which decreases in size posteriorly. Where the full number is present, it is a very general rule that the first greatly exceeds the second in size; but in the present case the decrease is much more gradual. The crowns have the anterior part much elevated and divided into the three usual cusps. Instead, however, of the external one being much larger and higher than the other two, they are more nearly equal (figure 23). They are placed at the points of a triangle whose sides are subequal, the shortest side, or base, being directed inwards and a little forwards. Between the outer and anterior cusps a short more or less imperfect shear is developed, and a posterior shear is also present between the outer and inner cusps. The heel is composed of a central fore and aft secant ridge, on the inner side of which the border is slightly raised, thus giving the first step in the formation of the basin-shaped talon. The succeeding teeth are alike in structure, with the exception that the principal shear is proportionately less developed on account of the reduction in size of the anterior cusp of the trigon.

23

FIGURE 23.—Lower molars of *Oödectes herpestoides* Wortman; inside view; three times natural size. (Cotype.)

Of the superior dentition, figure 24, the single incisor, if it is correctly referred to this category, has a rather narrow, pointed crown, somewhat flattened upon its posterior surface, and corresponds most nearly with the second. The canine resembles that of the lower jaw, showing about an equal degree of stoutness and curvature. The first premolar is not preserved in any of the specimens. The second has an obtusely-pointed crown, with a small posterior basal cusp and an indistinct cingulum upon the inner margin of its base. The third is thicker, somewhat triangular in cross-section, with a more pronounced posterior basal cusp and a stronger internal cingulum. In many of the viverrines, more particularly the Herpestine and Paradoxurine sections, the third premolar has a tendency to develop an internal cusp, and the formation of the internal cingulum may be regarded as the initial stage in this process.

At first sight it would appear that the fourth superior premolar is of a too highly developed sectorial character to correspond with that of the lower jaw; but, curious as this may seem, there can be no possible doubt of the association. This tooth, however, betrays its primitive character in the large size

of its antero-external cusp, which is relatively high and conical, with its posterior portion little drawn out into a cutting surface. The posterior cusp is sharp and blade-like, but proportionally small. The two are separated by the usual vertical fissure. There is a distinct anterior basal cusp, which has an unusually external position. The presence of this cusp is a very constant feature of the viverrines, and its external position recalls the Herpestine section of the family. The internal cusp is of only moderate proportions and is placed well forwards, as in the civets in general.

The first molar is symmetrical in respect to the extension of the external angles; they are both equally extended and the two sub-equal external cusps are placed well inwards from the

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FIGURE 24.—Right upper jaw of *Oödectes herpestoides* Wortman; crown view; three times natural size. (Type.)

crown margin. The internal cusp is large and lunate, and there is a well-developed anterior and a very faint posterior intermediate cusp. The second molar has the antero-external angle well extended, but the posterior is short; with the exception of this difference its structure is very like that of the first. The third molar is not preserved, but the alveoli indicate not only its existence but its goodly size as well.

Vertebræ.—While the vertebral column is by no means complete, yet a number of the vertebræ are preserved and serve to give some idea of this part of the skeleton. A portion of the atlas, figure 25, shows that the perforations for the vertebral artery are the same as those in the civets. The body of the axis is rather long, sharply keeled below, and there is a well-developed peg-like odontoid process. The bodies of the remaining cervicals are short, depressed, and keeled; they are smaller in proportion to the size of the lumbars than in any of the living civets. The lumbars increase in size

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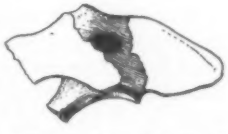

FIGURE 25.—Portion of atlas of *Oödectes herpestoides* Wortman; top view; three halves natural size. (Type.)

progressively backwards, showing elongated inferiorly keeled centra. The third lumbar, figure 26, has a spine of moderate height, and simple, cylindrical posterior, and hollow, half cylinder-like anterior zygapophyses, and distinct anapophyses and metapophyses. There is apparently no trace of the double tongue and groove pattern of certain of the contemporary Creodonts. The sacrum is composed of three anchylosed vertebræ, and there was a long and powerful tail.

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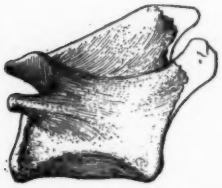

FIGURE 26.—Third lumbar vertebra of *Oödectes herpestoides* Wortman; side view; three halves natural size. (Type.)

Fore limb.—The scapula is represented by the proximal end in a damaged condition. The glenoid cavity is elliptical and cup-shaped; the neck is very short, the unusually heavy spine rises close to the glenoid border, as in the Binturong, and there was a prominent metacromion present.

The humerus, figure 27, is complete, but considerably crushed laterally; it exhibits the following characters: The head is well rounded, pyriform, and overhangs the axis of the shaft posteriorly; the greater tuberosity is inconspicuous, and does not reach the level of the head; the deltoid crest is large and extends well down the shaft; the distal extremity is broad, with large supinator ridge and internal condyle, and there is an entepicondylar foramen.

In its proximal portion, the ulna, figure 29, exhibits some peculiarities of structure to which that of the Binturong makes a very decided approach. The chief peculiarity is seen in the upward curvature of the under surface of the olecranon and its great lateral breadth. Upon its inner or radial side, it is produced in such a way as to form a broad shelf-like projection, as in the Binturong; it is relatively short and thick as in this latter species, in marked contrast with its elongated form in many of the contemporary Creodonts. The posterior wall of the greater sigmoid cavity has comparatively little elevation, giving to the cavity a shallow appearance, but the anterior boundary or coronoid process is prominent and well extended upon its radial side. Just in front of this latter process is seen the deep muscular impression for the attachment of the tendon of the anterior brachial muscle. The shaft is considerably flattened from side to side and traversed by broad shallow longitudinal grooves, which continue to the distal end. In the lower fourth of its extent, the shaft becomes sharply triangular in cross-section by reason of the development of a sharp ridge from the more or less rounded internal surface. This

ridge is very highly developed in the Binturong and less so in *Herpestes*. The distal end is not preserved.

The radius, figure 28, as compared with the ulna in size, holds about the same relationship as that seen in the civets in general. The head is cup-shaped and has an imperfectly circular outline, indicating thereby complete power of rotation.

FIGURE 27.—Humerus of *Oödectes herpestoides* Wortman; front view.
FIGURE 28.—Radius of same species; front view.
FIGURE 29.—Ulna of same species; front view.
All figures are three halves natural size. (Type.)

The shaft is slightly curved, somewhat compressed, and distinctly trihedral in its lower fourth. The distal end is provided with a well-developed styloid, deep tendinal grooves, and a well-excavated articular surface for the scaphoid and lunar.

The manus, figure 30, is sufficiently preserved to furnish a fair idea of its organization. All the carpal bones are present with the exception of the lunar, magnum, and pisiform. The scaphoid, centrale, trapezium, and trapezoid, with the proximal portions of the first three metacarpals attached, were found in the position shown in the accompanying figure, and there is reason to believe that the positions of the bones are substantially correct. The scaphoid is relatively large, and is articulated distally with the trapezium, trapezoid, and centrale. The trapezium is of moderate size and supports the pollex in the usual way, which is not to any very great degree opposable, if at all. The trapezoid is rather large, of an imperfectly triangular form, and supports the second metacarpal. The centrale may be said to have an outline intermediate between a quadrate and triangular pattern; it rests unequally upon trapezoid and magnum, and lies under the junction of the scaphoid and lunar. The unciform resembles that of the modern civets, and presents a lateral facet for articulation with the lunar. The cuneiform is flattened from above downwards, and articulated with the unciform in quite the usual way. Of the pollex is the heaviest, the

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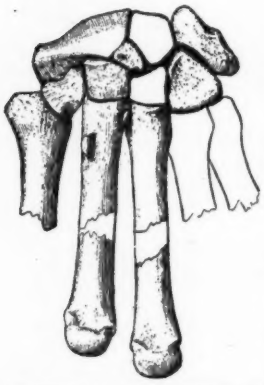


FIGURE 30. — Left manus of *Oödestes herpestoides* Wortman; two and one-fourth times natural size. (Type.)

Of the metacarpals, that second and third being more slender, with rather narrow proximal extremities. On the upper portion of the shaft of the second is seen a prominent tuberosity, and a less conspicuous one upon the third near the proximal extremity. These bony protuberances, if normal, probably served for the attachment of the long and short carpal extensors, since, in position, they correspond nearly to the insertion of these muscles. The length of the metapodials cannot be ascertained, but they were presumably short. Their distal ends are hemispherical and keeled like those of the modern civets. Some fragments of the proximal row of phalanges are preserved, and these are again like those of *Herpestes* and the Binturong. One perfect phalanx, figure 31, belonging probably to the second row of the fore foot, is long and slender, notably more so than in the Binturong, wherein they are longer than usual in the Viverridæ. No ungual phalanges are known.

[To be continued.]

ART. XVI.—*The Electromagnetic Effects of Moving Charged Spheres;* by EDWIN P. ADAMS.

THE magnetic effects due to moving charges were first shown experimentally by Professor Rowland* in 1876. Dr. E. Lecher,† in 1884, thinking that the importance of the experiment rendered its repetition desirable, did so, but with negative results. Insufficient data regarding his experiment make it difficult to point out the cause of his failure to obtain the effect. Since then, Professor Rowland's results have been fully confirmed by Professor W. C. Röntgen‡ in 1885; by Rowland and Hutchinson§ in 1889; and by Professor F. Himstedt‖ later in the same year.

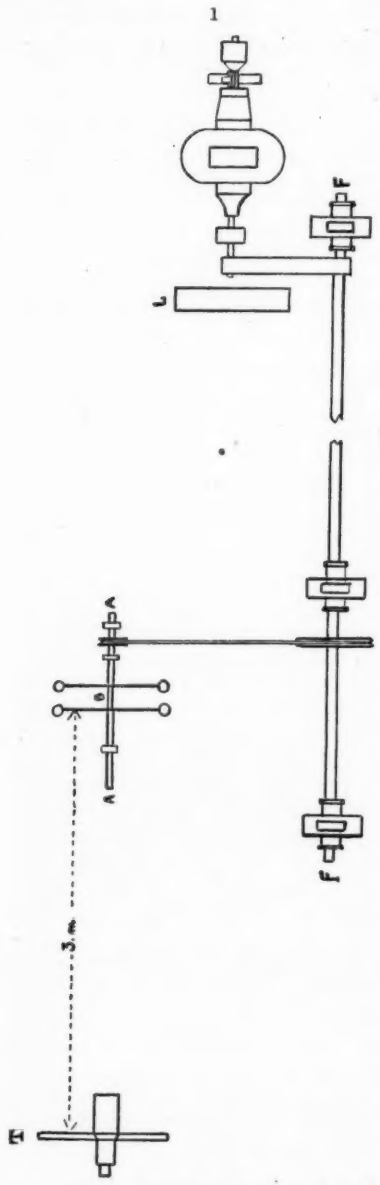
The next experimental attack upon this problem was by M. V. Cremieu¶ at Paris in 1900–1. His experiments were originally undertaken to determine whether a changing magnetic field exerts a mechanical force upon an electrically charged body. The negative results obtained led him to undertake a series of experiments on the magnetic effect of moving electric charges. The results of these experiments are apparently all opposed to the results obtained by the above observers. The data which have thus far appeared do not give sufficient details to render it certain that positive results should have been expected. Cremieu himself states that he is convinced that the effect does not exist.

The great importance of the experiment would seem to render further investigation desirable, and the present paper contains a description of an experiment with this end in view.

All previous experiments have been made with rotating disks. With one exception the direct effect of a charged rotating disk upon a magnetic needle has been examined. The exception referred to is the method employed by Cremieu in one of his experiments where he sought to observe the inductive effect of charging and discharging a rotating disk upon a neighboring coil of wire in circuit with a sensitive galvanometer.

The use of charged spheres was suggested by Professor J. J. Thomson** in 1881, and he gives a calculation of the mag-

* This Journal (3), xv, p. 30, 1878.
† Rep. d. Phys., xx, p. 151, 1884.
‡ Sitz. d. Berlin Akad., p. 198, 1885.
§ Phil. Mag. (3), xxvii, p. 445, 1889.
‖ Wied. Ann., xxxviii, p. 560, 1889.
¶ Comptes Rendus, cxxx, p. 1544; cxxxi, p. 578; cxxxi, p. 797; cxxxii, p. 327.
** Phil. Mag., xi, p. 236, 1881.

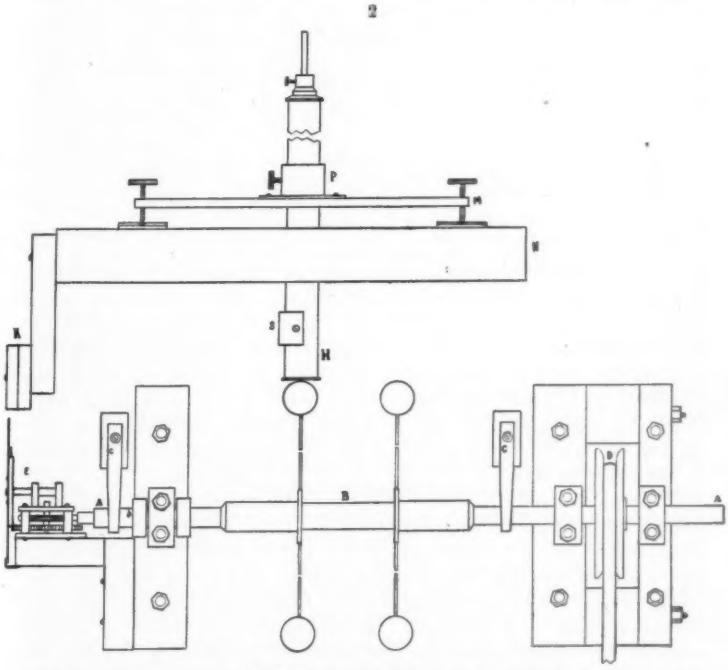


netic force which would be produced by the motion of a sphere charged to the highest possible potential. In many respects this seems the most natural method of procedure, and was adopted in this experiment. The description of the apparatus employed follows.

A hollow brass shaft, AA (figs. 1 and 2), is separated into two portions by the hard wood bar B. The shaft turns in fiber bearings, and the pulley and belt at D, fig. 2, communicate power from the countershaft FF. The spheres which carry the electric charges are spun out of sheet copper into hemispheres, soldered together. There are two sets of spheres, 16 in each set. Brass rods pass through the hollow spheres, and are soldered to them. These brass rods are screwed into collars carried on the axle. The two sets of spheres are thus insulated from each other by the hard wood bar, 8^{cm} in length. The electricity is communicated to the two portions of the axle by the copper brushes CC, fig. 2, and the spheres, being in contact with the axle, thus become charged themselves.

The speed-counter E, fig. 2, is directly attached to one end of the axle. The gearing is so proportioned that one revolution of the crown-gear and dial I, corresponds to 500 revolutions of the axle.

The magnetic system upon which the direct effect of the moving charged spheres is observed, is enclosed in the brass tube H, closed on the bottom by a glass plate coated with tin-foil. This tin-foil is cut into strips parallel to the axle for the purpose of preventing conduction currents flowing in it in a direction in which they could cause a deflection of the needle. The needle-system is carried on a piece of mica, 7·5^{cm} long and 1·25^{cm} wide. Pieces of well-hardened magnetized watch-



spring are cemented at the lower edge and near the upper edge of the mica so as to form an astatic system. The needles are placed perpendicularly to the axle. The mirror is also cemented to the mica, a little above its center, and is observed through an opening in the brass tube at S, covered with a thin glass plate. The whole needle-system is suspended by a quartz fiber, 32^{cm} in length.

The magnetometer tube H is carried in a brass collar P, which is screwed to the brass plate M. This plate is provided with levelling screws, and rests on the board shelf N, supported at its ends on two brick piers. Underneath the levelling screws are placed pieces of felt, to take up any mechanical

vibrations. By means of control magnets placed on the brass plate M any required degree of sensitiveness can be given to the needle-system. All metallic parts of the magnetometer tube and supports are earthed. A single turn of wire at K serves to determine the needle-constant. The deflections are read by means of the telescope and scale T, placed at a distance of 3 meters from the mirror.

Power for driving the spheres is furnished by a 4 horse-power motor, at a distance of 7 meters from the magnetometer. A heavy iron casting, L, in front of the motor, gives additional screening of magnetic disturbances due to the motor. The motor is belted to the steel countershaft, FF, which turns in hangers placed along the cement floor. A rigid wooden framework is built up from the floor to carry the revolving spheres. The axle AA is at a distance of 1 meter from the floor. The brick piers which support the magnetometer are entirely separate from the floor and from the framework which carries the spheres.

The electricity for charging the spheres is furnished by the battery of 10,000 storage cells used by Professor Trowbridge in investigations in spectrum analysis, etc., and which has been described by him in this Journal. The battery is on the third floor of the laboratory, while the apparatus for this experiment was set up in the basement. The wires leading down from the battery are well separated from each other and from surrounding walls, except for a short distance, where they are carried in thick-walled glass tubes. A commutator is inserted to reverse the sign of the charges of the spheres.

A good deal of difficulty was met with due to the wind produced by the revolving spheres. These have a velocity of about three miles a minute, and the wind was sufficient to shake the shelf supporting the magnetometer so that it was impossible to take readings owing to the continual vibration of the needle-system. This made it necessary to build a shield around the spheres to keep the wind from blowing directly on any part of the magnetometer support. In order to bring the needles as close as possible to the spheres, the top board of the shield has a hole cut in it into which the lower end of the magnetometer tube is placed. The hole is closed on the bottom by a very thin glass plate, which the spheres just clear when revolving.

Not the slightest movement of the needle could be observed when the motor alone was run, or when the motor and steel-shaft along the floor were run. But the cutting of the earth's magnetism by the brass axle was sufficient to produce a deflection of several centimeters. As long as the speed remained perfectly constant this gave no trouble. But if the speed

varied by even a small amount this was very troublesome, and was one of the principal sources of error in the experiment.

The two sets of spheres are charged oppositely, one being connected to the positive pole of the battery and the other to the negative pole. On charging them while at rest a very small deflection was observed. This was not due to direct electrostatic effect, but to the rush of current flowing in to charge the spheres. It is only an instantaneous effect, the needle coming back to its original position of equilibrium almost immediately. This deflection was entirely gotten rid of by inserting a large water-resistance in series with the battery. With this resistance in, and the spheres at rest, no effect could be observed on the needle when the spheres were charged, or when the charge was reversed in sign.

When the spheres were set revolving, and the electrification was reversed, a distinct deflection of the needle was produced. It was difficult always to get satisfactory readings of this deflection due to slight changes in the speed and the consequent change of the zero-point. But the qualitative effect was unmistakable. The deflection was in the direction to be expected; that is, a positively charged sphere gives rise to a magnetic force in the same sense as a current flowing in the direction of motion.

At first sight there seems to be a possible alternative explanation of this effect. It may be that the charges are continually swept off from the rapidly moving spheres by their motion through the air, and that thus a continual flow of electricity is produced in the wires which connect the battery to the spheres. But the strength of such a current, if it existed, would be far too small to produce the observed effects owing to the very large resistance of the water in series with the battery. With this consideration, and the absolute regularity with which the effect was observed, there appears to be no reasonable doubt that this is an actual magnetic effect due to moving charges.

All observations were made between the hours of one and five in the morning. It was impossible to get any satisfactory readings in the day time, owing mainly to the magnetic disturbances produced by the electric cars. Furthermore, the speed of the motor was more constant at that time than during the day, since the load on the mains of the power-plant varied less. It was found most satisfactory to have the sensitiveness of the needle such that a deflection of 5–15 millimeters was produced on reversing the electrification. Much greater sensitiveness could easily have been obtained, but the zero-point varied so much that the readings were less reliable. The speed used was about 50 revolutions per second. At this speed no trouble was experienced from wind or from vibrations commu-

nicated directly to the magnetic system. But when the speed exceeded 60 revolutions per second the needle began to vibrate sufficiently to make observations difficult.

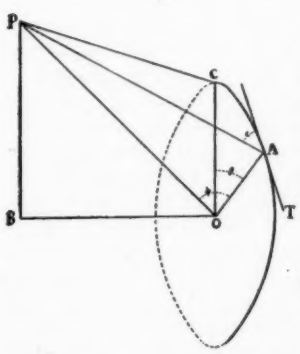
The following is the method used in taking a series of readings. The motor was started, and the time of 2500 revolutions of the axle determined. The spheres were then charged; two elongations of the needle on one side of the zero-position were read, and one on the other side; the electrification was then reversed, and similar readings taken. Sometimes the readings were not taken until the needle had come nearly to rest, and its new equilibrium position estimated. This was repeated until about ten reversals had been made. The speed was then again determined. The average of the deflections, and the average of the two values of the speed were taken as representing the series. The needle constant was determined before and after a number of series.

An attempt will be made in the following to compare the results obtained with the results expected from theory. From reasons which will appear later this comparison can be regarded only as approximate, and is given merely to show that the observed results are of the right order of magnitude. Rowland's method of using the experiment to determine V, the ratio of the units, will be used.

The magnetic force produced by a moving charge, q, traveling at velocity v, is:*

$$H = \frac{qv \sin \epsilon}{\rho^2}$$

where ρ is the radius vector drawn from the charge to the point at which H is measured, and ϵ is the angle between ρ and the direction of motion. This expression holds only in case the velocity v is small compared with the velocity of light. This may also be taken as the magnetic force produced by a moving charged sphere, the charge being supposed concentrated at its center. The force acts in a direction perpendicular to ρ and to the direction of motion.



The magnetic force at either the upper or lower needle, due to one of the spheres at any point in its path, is found as follows:

The two sets of spheres revolve

* J. J. Thomson, Phil. Mag., xi, p. 236, 1881; Heaviside, Electrical Papers, vol. ii, p. 505.

in two parallel circles, distant b from each other. The plane of revolution is taken perpendicular to the plane of the paper (fig. 3). The needles lie in one of the planes of revolution. The force at P due to the sphere at A is required.

ρ = PA.
b = OB.
d = PB.
c = OC = OA = radius of revolution.
ϵ = angle between ρ and tangent at A.
θ = angle between vertical radius and radius to A.

$$\rho^2=d^2+b^2+c^2-2c\sqrt{d^2+b^2}\cos\phi$$

$$\cos\phi=\cos\theta\frac{d}{\sqrt{d^2+b^2}}$$

$$\rho^2=d^2+b^2+c^2-2dc\cos\theta$$

$$\cos\epsilon=\frac{d\sin\theta}{\rho}$$

$$\sin\epsilon=\sqrt{\frac{(d\cos\theta-c)^2+b^2}{d^2+b^2+c^2-2dc\cos\theta}}$$

The force acts in a direction perpendicular to ρ and the tangent at A. The component of this force in the direction of the normal to the plane of revolution is required. Let ψ be the angle between the direction of the force and the normal to the plane of revolution.

$$\cos\Psi=\frac{d\cos\theta-c}{\sqrt{(d\cos\theta-c)^2+b^2}}$$

$$v=2\pi c\mathrm{N}$$

where N is the number of revolutions per second.

Hence:

$$\mathrm{X}=\frac{2\pi\mathrm{N}cq\,(d\cos\theta-c)}{\mathrm{V}[d^2+b^2+c^2-2dc\cos\theta]^{\frac{3}{2}}}$$

X is the component of the force at P in the direction of the axle due to the sphere at A. V is the ratio of the units. The capacity of the spheres and their potential are measured in electrostatic units.

Figure 4 is plotted from this expression, and shows how the force varies with the position of the spheres. The upper curve gives the resultant force at the lower needle due to both sets of spheres, and the lower curve, which is nearly a straight line, gives the force at the upper needle. Let the mean value of

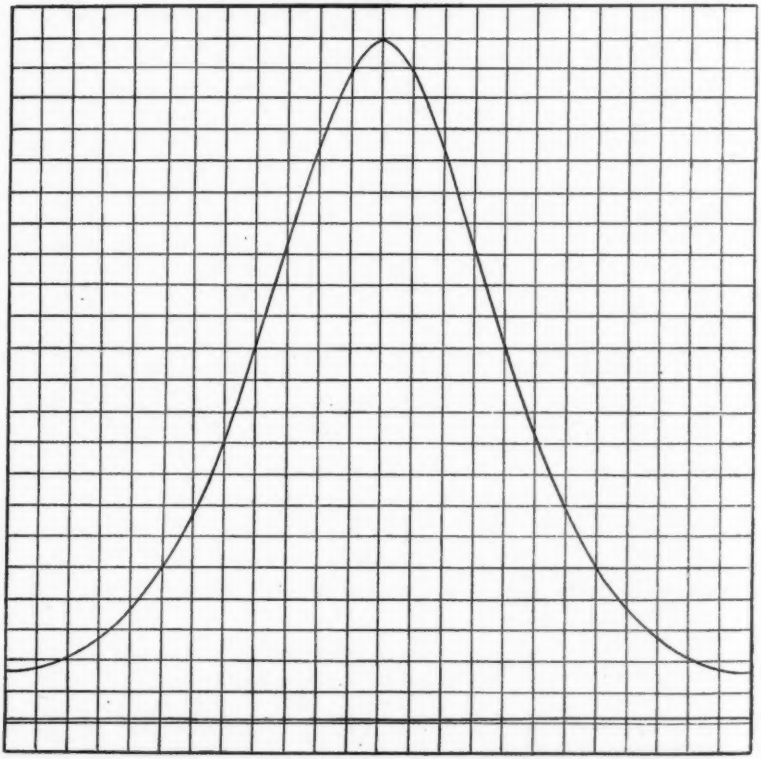
the force at the lower needle obtained by time-integration of the curve be:

$$\frac{2\pi N q}{V} A$$

and the mean value of the force at the upper needle:

$$\frac{2\pi N q}{V} B$$

4



Then the effect on the needle will be the same as if constant forces of these magnitudes acted upon it. The same result could be obtained by imagining two coils of wire passing through the centers of the two sets of spheres through which a current was sent in opposite directions of such a magnitude that the same amount of electricity passed any point per second as in the case of the charged spheres.

Force at lower needle due to calibrating coil:

$$\frac{2\pi \mathrm{I}h^3}{(h^2+x^2)^{\frac{3}{2}}}=2\pi \mathrm{IC}$$

h being the radius of the coil, and x its distance from the plane of the needles.

Force at upper needle due to calibrating coil:

$$2\pi \mathrm{I}\left[\frac{1}{r}\left(\frac{h}{r}\right)^2 \mathrm{P}_2(\cos\theta)-\frac{1}{r}\,\frac{3}{2}\left(\frac{h}{r}\right)^4 \mathrm{P}_4(\cos\theta)+\ldots\ldots\right]=2\pi \mathrm{ID}$$

r being the distance of the center of the coil to the upper needle, and θ the angle between the axis of the coil and r.

Let M be the moment of the lower needle, and H the earth's horizontal magnetic force at its center; M′ and H′ the corresponding values for the upper needle. Let θ be the angular deflection of the needle-system produced by the current in the calibrating coil, and ϕ the angular deflection produced by the moving charged spheres.

Equating the couple acting on the needle-system due to the earth's field to the couple acting on the needle-system due to the current in the calibrating coil, and putting $\mathrm{M}/\mathrm{M}'=1$, we have:

$$\frac{\mathrm{HM}-\mathrm{H}'\mathrm{M}'}{\mathrm{M}}=\frac{2\pi \mathrm{I}(\mathrm{C}-\mathrm{D})}{\tan\theta}$$

Similarly, equating the couple acting on the needle-system due to the earth's field to the couple acting on the needle-system due to the revolving charged spheres, we have:

$$\frac{\mathrm{HM}-\mathrm{H}'\mathrm{M}'}{\mathrm{M}}=\frac{2\pi \mathrm{N}q(\mathrm{A}-\mathrm{B})}{\mathrm{V}\tan\phi}$$

Hence

$$\mathrm{V}=\frac{\mathrm{A}-\mathrm{B}}{\mathrm{C}-\mathrm{D}}\,\frac{\mathrm{N}q}{\mathrm{I}}\,\frac{\tan\theta}{\tan\phi}$$

Let δ be the scale deflection on reversing the current I in the calibrating coil, and Δ the scale deflection on reversing the charges of the spheres. Then:

$$\mathrm{V}=\frac{\mathrm{A}-\mathrm{B}}{\mathrm{C}-\mathrm{D}}\,\frac{\mathrm{N}q}{\mathrm{I}}\,\frac{\delta}{\Delta}$$

d and d', the distances of the centers of the lower and upper needles respectively from the axle, were determined by means of a cathetometer, the distance of the mirror from the center of the axle being directly measured, from which d and d' were obtained.

The current sent through the calibrating coil for determining the needle-constant was measured by a Weston milliammeter.

The value found was accurate to at least one-half of one per cent. which is sufficient for this purpose.

The charge of the spheres is the most uncertain element in the quantitative determination, and it is this uncertainty especially which makes the method of revolving spheres far less suitable for quantitative work than the method of rotating disks, particularly as employed by Rowland in his second experiment. If a single set of spheres had been used, charged to the same potential, an equal opposite charge would have been induced on neighboring conductors which would have travelled with the charges on the spheres. It would have been difficult to determine just what the resultant effect should be. For this reason two distinct sets of spheres were used, charged oppositely, the spheres always keeping the same relative positions. It was then assumed that the only moving charges were those carried on the moving spheres. The capacity of the spheres was calculated on the assumption that they were the only conductors present. The charge on any one sphere was calculated by the method of images, the charges on all the other spheres being regarded as concentrated at their centers. We then have (Maxwell, volume i, section 159):

$\pm q$ = charge of each sphere.
$\pm$ P = its potential.
a = its radius.
f_1, f_2, etc. = the distances of the centers of the spheres from one another.

Then the charge on any sphere is given by:

$$q = Pa - qa\left(\frac{1}{f_1} + \frac{1}{f_3} + \ldots\right) + qa\left(\frac{1}{f_2} + \frac{1}{f_4} + \ldots\right)$$

where the odd subscripts refer to spheres of the same set and even subscripts to spheres of the other set.

Numerous determinations have shown that the potential of the individual cells making up the battery averages almost exactly two volts when freshly charged. On this basis, assuming perfect insulation, the potential used was 20,000 volts. It was thought best, however, to get a closer estimate of the potential, since the insulation was not perfect. This was measured by means of a guard-ring electrometer. If the radius of the movable disk is R, the inside radius of the guard-ring R′, D the distance between the moveable disk and the fixed disk, W the weight required to balance the attraction of the two disks, and g the acceleration of gravity,*

* Maxwell, volume i, section 217.

$$P = 4D\sqrt{\frac{gW}{R^2 + R'^2}}$$

$$R = 5{\cdot}933$$
$$R' = 6{\cdot}033$$
$$g = 980{\cdot}$$

The following determinations were made:

D	G	P
3·0	2·078	64·0
2·5	2·780	62·0
2·0	4·505	63·0

These values of the potential are in electrostatic units. To convert into volts multiply by 300, and the potential as measured by the electrometer is 18,900.

As a further check the maximum sparking distance between two metallic spheres was measured. With polished brass spheres, 2·6^{cm} in diameter, this distance was found to be 0·58^{cm}. According to the observations of Baille,* this corresponds to a potential:

$$P = 63$$

Baille's observations were with spheres of different sizes. In the region of a spark-length of this magnitude, he found that spheres of 3^{cm} diameter gave nearly the same results as spheres 1^{cm} in diameter, so that his results for spheres of 3^{cm} diameter can be used with very small error for spheres 2·6^{cm} in diameter.

From these determinations, the value

$$P = 63$$

in electrostatic units is taken as the potential of the spheres. The great advantage in using a storage battery as the source of electricity is that one measurement is sufficient to determine the potential. The battery when used was always freshly charged, and the variations in its potential from one experiment to another were very small.

Substituting the following numerical values:

Radius of spheres	= 1·35
c	= 20·38
d	= 22·91
d'	= 29·28
b	= 8·16
x	= 19·30
h	= 2·55

we find

A–B	= 1·541
C–D	= 0·000232
q	= 1·38 × 63

* J. J. Thomson, Recent Researches, p. 77.

Below are given the successive equilibrium positions of the needle in six series of reversals, as an illustration of the results that have been obtained:

No. 1.		No. 2.		No. 3.	
	63		165		130
60	66	172	178	109	122
64	76	188	223	122	130
71	79	200	220	112	130
69	76	213		121	140
69				146	156
				164	171
				162	

The needles were then moved a little closer to the spheres, giving:

$$d = 22{\cdot}47$$
$$d' = 28{\cdot}84$$
$$A\text{-}B = 1{\cdot}834$$

No. 4.		No. 5.		No. 6.	
92	114		164	108	84
106	135	148	156	113	118
120	135	148	149	108	155
142	149	152	143	125	135
164	192	139	155		
		147			

In all these cases, the first column gives the readings when the spheres directly under the magnetometer are positively charged, and the second column when they are negatively charged. In No. 1, the successive deflections are: 3, 6, 2, 12, 5, 8, 10, 7, 7; giving an average of 6·7. In No. 4 they are: 21, 13, 0, 8, 18, 18, 9, 19, –6, 10, –8, 7, 9; giving an average of 9. It is evident that with variations such as are present here the measurements can be regarded only as a rough approximation. But the most important fact to be observed is that when the spheres directly under the magnetometer are changed from negative to positive a deflection toward the small figures of the scale takes place; and when changed from positive to negative a reverse deflection takes place. When on account of changes in the zero-point some of the deflections do not apparently follow this rule, they are entered with a negative sign in finding the average deflection. There has invariably resulted a positive deflection on taking the average.

Earlier experiments were made with four spheres in each set. With this number it was possible to get much higher speeds—75–85 revolutions per second. Similar qualitative results were observed, but when comparison with theory was attempted it appeared that fair agreement could be obtained if

the maximum value of the magnetic force were used instead of the average. This was largely an accidental result, a sufficient number of reversals not having been made.

It would have been desirable to have made a larger number of reversals in each series, but after the apparatus had been running for some time the bearings heated so much that it was impracticable.

The following table gives the values of the ratio of the units obtained from the above readings:

No	N	A	I	δ	V	
1	42	6·7	·00364	26	2·6	10^{10}
2	55	10·6	·00355	31	2·6	
3	55	9·0	·00355	31	3·1	
4	49	11·3	·00298	29	2·9	
5	41	5·5	·00280	15	2·7	
6	48	7·0	·00280	15	2·6	
		Average			2·8	10^{10}

These results may be taken as fairly representing all that have been obtained. The agreement between theory and experiment is fully as good as could be expected when all the uncertain elements in the determination are taken into consideration.

These uncertain elements are: (1) The actual charges carried by the spheres and the effect of surrounding bodies, especially the plate coated with tin-foil covering the lower end of the magnetometer tube. This tin-foil is cut into strips about 1 millimeter in width and its effect must be small. (2) The non-uniformity of distribution of electricity upon the spheres so that the charges cannot be regarded accurately as concentrated at their centers. (3) Errors in reading the deflection of the needle due to outside disturbances.

Experiments have also been made with the direction of motion of the spheres reversed. The results obtained are similar in every respect to those given above, except that they are reversed. Experiments using only a portion of the 10,000 cells of the storage battery gave results which agree fairly well with the preceding. The deflections were too small, however, to expect very close agreement.

Jefferson Physical Laboratory, Harvard University, Cambridge, Mass.

ART. XVII.—*The Nadir of Temperature and Allied Problems;* by JAMES DEWAR, LL.D., F.R.S. [Bakerian Lecture (abstract) read before the Royal Society,* June 13, 1901.]

1. Physical Properties of Liquid and Solid Hydrogen. 2. Separation of Free Hydrogen and other Gases from Air. 3. Electric Resistance Thermometry at the Boiling Point of Hydrogen. 4. Experiments on the Liquefaction of Helium at the Melting Point of Hydrogen. 5. Pyroelectricity, Phosphorescence, etc.

Details are given in this paper which have led to the following results:—

The helium thermometer which records 20°·5 absolute as the boiling point of hydrogen, gives as the melting point 16° absolute. This value does not differ greatly from the value previously deduced from the use of hydrogen gas thermometers, viz., 16°·7. The lowest temperature recorded by gas thermometer is 14°·5, but with more complete isolation and a lower pressure of exhaustion, it will be possible to reach about 13° absolute, which is the lowest temperature that can be commanded by the use of solid hydrogen. Until the experiments are repeated with a helium gas thermometer filled at different pressures, with the gas previously purified by cooling to the lowest temperature that can be reached by the use of solid hydrogen, no more accurate values can be deduced.

The latent heat of liquid hydrogen about the boiling point as deduced from the vapor pressures and helium-thermometer temperatures, is about 200 units, and the latent heat of solid hydrogen is about 16 units.

The order of the specific heat of liquid hydrogen has been determined by observing the percentage of liquid that has to be quickly evaporated under exhaustion in order to reduce the temperature to the melting point of hydrogen, the vacuum vessel in which the experiment is made being immersed in liquid air. It was found that in the case of hydrogen the amount that had to be evaporated was 15 per cent. This value, along with the latent heat of evaporation, gives an average specific heat of the liquid between freezing and boiling point of about 6. When liquid nitrogen was similarly treated for comparison, the resulting specific heat of the liquid came out 0·43 or about 6 per atom. Hydrogen therefore follows the law of Dulong and Petit, and has the greatest specific heat of any known substance.

The same fine tube used in water, liquid air, and liquid hydrogen gave respectively the capillary ascents of 15·5, 2 and 5·5 divisions. The relative surface tension of water, liquid air, and liquid hydrogen are therefore in the proportion of 15·5, 2, 0·4. In other words, the surface tension of hydrogen at its

* From an advance proof received from the author.

boiling point is about one-fifth that of liquid air under similar conditions. It does not exceed one thirty-fifth part the surface tension of water at the ordinary temperature.

The refractive index of liquid hydrogen, determined by measuring the relative difference of focus for a parallel beam of light sent through a spherical vacuum vessel filled in succession with water, liquid oxygen, and liquid hydrogen, gave the value 1·12. The theoretical value of the liquid refractive index is 1·11 at the boiling point of the liquid. This result is sufficient to show that hydrogen, like liquid oxygen and nitrogen, has a refractivity in accordance with theory.

Free hydrogen, helium, and neon have been separated from air by two methods. The one depends on the use of liquid hydrogen to boil the dissolved gases out of air kept at a temperature near the melting point of nitrogen; the other on a simple arrangement for keeping the more volatile gases from getting into solution after separation by partial exhaustion. By the latter mode of working something like 1/34000th of the volume of the air liquefied appears as uncondensed gas. The latter method is only a qualitative one for the recognition and separation of a part of the hydrogen in air. In a former paper on the "Liquefaction of Air and the Detection of Impurities,"* it was shown that 100 cc. of liquid air could dissolve 20 cc. of hydrogen at the same temperature. The crude gas separated from air by the second method gave on analysis—hydrogen 32·5 per cent., nitrogen 8 per cent., helium, neon, &c., 60 per cent. After removing the hydrogen and nitrogen the neon can be solidified by cooling in liquid hydrogen and the more volatile portions separated.

There exists in air a gaseous material that may be separated without the liquefaction of the air. For this purpose air has to be sucked through a spiral tube filled with glass wool immersed in liquid air. After a considerable quantity of air has been passed, the spiral is exhausted at the low temperature of the liquid air bath. The spiral tube is now removed and allowed to heat up to the ordinary temperature, and the condensed gas taken out by the pump. After purification by spectroscopic fractionation the gas filled into vacuum tubes gives the chief lines of xenon. The spectroscopic examination of the material will be dealt with in a separate paper by Professor Liveing and myself. A similar experiment made with liquid air kept under exhaustion, the air current allowed to circulate being under a pressure less than the saturation pressure of the liquid to prevent liquefaction, resulted in crypton being deposited along with the xenon.

A study of fifteen electric-resistance thermometers as far as the boiling point of hydrogen, has been made, and the results

* Chem. Soc. Proc., 1897.

reduced by the Callendar and Dickson methods. The table [here omitted] gives the results for seven thermometers, viz., two of platinum, one of gold, silver, copper, and iron, and one of platinum-rhodium alloy. Of these the lowest boiling point for hydrogen was given by the gold thermometer. Next to it came one of the platinum thermometers, and then silver, while copper and the iron differ from the gold value by 26 and 32 degrees respectively. The gold thermometer would make the boiling point 23°·5 instead of the 20°·5 given by the gas thermometer. Then the reduction of temperature under exhaustion amounts to only 1° instead of 4° as given by the gas thermometer. The extraordinary reduction in resistance of some of the metals at the boiling point of hydrogen is very remarkable. Thus copper has only 1/105th, gold 1/30th, platinum 1/35th to 1/17th, silver 1/24th the resistance at melting ice, whereas iron is only reduced to 1/8th part of the same initial resistance. The real law correlating electric resistance and temperature within the limits we are considering is unknown, and no thermometer of this kind can be relied on for giving accurate temperatures up to and below the boiling point of hydrogen. The curves are discussed in the paper, and I am indebted to Mr. J. H. D. Dickinson and Mr. J. E. Petavel for help in this part of the work.

Helium separated from the gas of the King's Well, Bath, and purified by passing through a U-tube immersed in liquid hydrogen, was filled directly into the ordinary form of Cailletet gas receiver used with his apparatus, and subjected to a pressure of 80 atmospheres, while a portion of the narrow part of the glass tube was immersed in liquid hydrogen. On sudden expansion from this pressure to atmospheric pressure a mist from the production of some solid body was clearly visible. After several compressions and expansions, the end of the tube contained a small amount of a solid body that passed directly into gas when the liquid hydrogen was removed and the tube kept in the vapor of hydrogen above the liquid. On lowering the temperature of the liquid hydrogen by exhaustion to its melting point, which is about 16° absolute, and repeating the expansions on the gas from which the solid had separated by the previous expansions at the boiling point or 20°·5, *no mist was seen.* From this it appears the mist was caused by some other material than helium, in all probability neon, and when the latter is removed no mist is seen, when the gas is expanded from 80 to 100 atmospheres, even although the tube is surrounded with solid hydrogen. From experiments made on hydrogen that had been similarly purified like the helium and used in the same apparatus, it appears a mist can be seen in hydrogen (under the same conditions of expansion as applied to the helium sample of gas) when the initial

temperature of the expanding gas was twice the critical temperature, but it was not visible when the initial temperature was about two and a-half times the critical temperature. This experience applied to interpret the helium experiments, would make the critical temperature of the gas under 9° absolute.

Olszewski in his experiments expanded helium from about seven times the critical temperature under a pressure of 125 atmospheres. If the temperature is calculated from the adiabatic expansion starting at 21° absolute, an effective expansion of only 20 to 1 would reach 6°·3, and 10 to 1 of 8°·3. It is now safe to say, helium has been really cooled to 9° or 10° absolute without any appearance of liquefaction. There is one point, however, that must be considered, and that is the small refractivity of helium as compared to hydrogen, which, as Lord Rayleigh has shown, is not more than one-fourth the latter gas. Now as the liquid refractivities are substantially in the same ratio as the gaseous refractivities in the case of hydrogen and oxygen, and the refractive index of liquid hydrogen is about 1·12, then the value for liquid helium should be about 1·03, both taken at their respective boiling points. In other words, liquid helium at its boiling point would have a refractive index of about the same value as liquid hydrogen at its critical point, and as a consequence, small drops of liquid helium forming in the gas near its critical point would be far more difficult to see than in the case of hydrogen similarly situated.

The hope of being able to liquefy helium, which would appear to have a boiling point of about 5° absolute, or one-fourth that of liquid hydrogen, is dependent on subjecting helium to the same process that succeeds with hydrogen; only instead of using liquid air under exhaustion as the primary cooling agent, liquid hydrogen under exhaustion must be employed, and the resulting liquid collected in vacuum vessels surrounded with liquid hydrogen. The following table embodies the results of experience and theory:—

Initial temperature.	Initial temperature.	Critical temperature.	Boiling points.
Liquid helium?	5°?	2°?	1°?
Solid hydrogen	15	6	4
Liquid "	20	8	5 (He?)
Exhausted liquid air	75	30	20 (H)
52° C	325	130	86 (Air)
Low Red Heat	750	304	195 (CO_2)

The first column gives the initial temperature before continuous expansion through a generator, the second the critical point of the gas that can be liquefied under such conditions, and the third the boiling point of the resulting liquid. It will

be seen that by the use of liquid or solid hydrogen as a cooling agent we ought to be able to liquefy a body having a critical point of about 6° to 8° absolute and boiling point of about 4° or 5° absolute. Then, if liquid helium could be produced with the probable boiling point of 5° absolute, this substance would not enable us to reach the zero of temperature; another gas must be found that is as much more volatile than helium as it is than hydrogen in order to reach within 1° of the zero of temperature. If the helium group comprises a substance having the atomic weight 2, or half that of helium, such a gas would bring us nearer the desired goal. In the meantime the production of liquid helium is a difficult and expensive enough problem to long occupy the scientific world.

A number of miscellaneous observations have been made in the course of this inquiry, among which the following may be mentioned. Thus the great increase of phosphorescence in the case of organic bodies cooled to the boiling point of hydrogen under light stimulation is very marked, when compared with the same effects brought about by the use of liquid air. A body like sulphide of zinc cooled to 21° absolute and exposed to light shows brilliant phosphorescence on the temperature being allowed to rise. Bodies like radium that exhibit self-luminosity in the dark, cooled in liquid hydrogen maintain their luminosity unimpaired. Photographic action is still active although it is reduced to about half the intensity it bears at the temperature of liquid air. Some crystals when placed in liquid hydrogen become for a time self-luminous, on account of the high electric stimulation brought about by the cooling causing actual electric discharges between the crystal molecules. This is very marked with some platino-cyanides and nitrate of uranium. Even cooling such crystals to the temperature of liquid air is sufficient to develop marked electrical and luminous effects. Considering that both liquid hydrogen and air are highly insulating liquids, the fact of electric discharges taking place under such conditions proves that the electrical potential generated by the cooling must be very high. When the cooled crystal is taken out of either liquid and allowed to increase in temperature, the luminosity and electric discharges take place again during the return to the normal temperature. A crystal of nitrate of uranium gets so highly charged electrically that, although its density is 2·8 and that of liquid air about 1, it refuses to sink, sticking to the side of the vacuum vessel and requiring a marked pull on a silk thread, to which it is attached, to displace it. Such a crystal rapidly removes cloudiness from liquid air by attracting suspended particles to its surface. The study of pyro-electricity at low temperatures will solve some very important problems.

SCIENTIFIC INTELLIGENCE.

1. *Magnetic effect of Electrical Convection.*—The magnetic effect produced by the motion of an electrified body, first proved by Rowland in 1876, and, though questioned by Cremieu, since then repeatedly confirmed (see the article by Edwin P. Adams in the present number, pp. 155–167, of this Journal), has been also established by HAROLD PENDER at the Johns Hopkins Physical Laboratory. An account of his experiments is given in a recent issue of the Johns Hopkins University Circular (No. 152, May–June, 1901). The method of Cremieu, which, however, gave him negative results, was employed. To produce the convection, two micanite discs (diam. 30 cm.) gilded on both sides and charged from a Voss machine and battery of six gallon Leyden jars, were driven at a speed of 75 to 100 revolutions per second. An interrupter made it possible to reverse the charge 12 to 25 times per second. Earth-connected condensing plates were fixed opposite each face of each disc and one centimeter distant. Between the two inside condensing plates was suspended a coil of 1295 turns of No. 21 copper wire connected through a commutating device with an extremely delicate astatic galvanometer. The coil, circuit, and galvanometer were enclosed in earth-connected metallic shields. The arrangement of the commutator was such that the alternating current induced in the coil by the reversal of charge in the rotating discs gave a steady deflection of the galvanometer.

The apparatus was used in two ways: (*a*) the two discs were rotated in the same direction and at any instant charged alike; and (*b*) the discs were rotated in opposite directions and charged oppositely at any instant; of these, the second method gave steadier deflections and was used more frequently. The direction of deflection, repeatedly tested, was found, as expected, to be always in accordance with Ampère's rule, i. e., the motion of a positive charge always produced the same effect as that of a conduction current flowing in the direction of motion of the charge.

The quantitative results are discussed, as follows: "The strength of a convection-current is defined as the quantity of electricity carried convectively past any point in unit time. On the assumption that a convection-current is magnetically equivalent to a conduction-current of the same strength, the current induced in the suspended coil on reversing the sign of electrification of the discs can be readily calculated from the dimensions of the apparatus and the difference of potential between the discs and plates. This calculated value of the current can then be compared with the observed value as deduced from the deflection of the galvanometer. The formula for the calculated value of the current involves the ratio of the two systems of electric units, so that instead of comparing directly the observed and calculated values of the current the two can be equated and the value of the ratio v thus determined. The value of this ratio thus found is a test for the accuracy of the assumption that a convection-current is equivalent magnetically to a

conduction-current. From 17 sets of observations, each set consisting of 18 separate determinations of the deflection and the other quantities involved, the mean value of v thus found was $3{\cdot}05\times10^{10}$, the determinations which differed from this the most being $2{\cdot}75\times10^{10}$ and $3{\cdot}24\times10^{10}$, respectively. The value of this constant is known to be $3{\cdot}00\times10^{10}$." Similar results were obtained when discs and condensing plates were divided each into six sectors by radial scratches, thus showing that the effect observed was not due to conduction-currents in their surfaces.

To prove, further, that the deflection observed was actually due to the magnetic action of the rotating charged discs, a further experiment was tried. The discs were rotated in the same direction and at any instant charged oppositely; and, again, rotated in opposite directions and at any instant charged alike. Their magnetic effects on the coil should then annul one another provided the two discs rotated with the same speed, and this was the result obtained. For example, when the two discs were rotating in same direction with speeds of $-86{\cdot}9$ and $-88{\cdot}8$ revolutions per second, respectively, and charged alike, the observed deflection was $-66{\cdot}2$; when they were rotating oppositely under the same conditions, with speeds $+88{\cdot}0$ and $-89{\cdot}6$, the observed deflection was $-1{\cdot}0$.

It is concluded, therefore, that the results obtained "show beyond any doubt *that electrical convection does produce magnetic action*, or, more exactly, that when the sign of electrification of a moving charged body is changed, an electric conduction-current is induced in a neighboring circuit, of a strength equal to that which would be induced in this circuit by reversing the direction of a conduction-current in a circuit coinciding with the path of the convection-current."

2. *American Association for the Advancement of Science.*—The fiftieth annual meeting of the American Association will be held in Denver, August 24 to 31. Professor C. G. Minot of Cambridge will preside as President. The address of the Permanent Secretary, Mr. L. O. Howard, is Washington, D. C. (Cosmos Club) until Aug. 15, after that, Brown Palace Hotel, Denver. Information in regard to transportation, hotel accommodation, etc., may be obtained from the Local Secretary, Mr. Arthur Williams, Chamber of Commerce, Denver.

OBITUARY.

Dr. Joseph LeConte, Professor of Geology and Natural History at the University of California, died on July 6, at the age of seventy-eight years. A notice is deferred until another number.

Professor Peter Guthrie Tait, the distinguished Scotch mathematician and physicist, died on July 4, at the age of seventy years. He had held the chair of Natural Philosophy in the University of Edinburgh since 1860. His original contributions, chiefly in mathematical physics, were numerous and important; he was also a lucid and suggestive writer in his treatment of scientific subjects from the popular standpoint.

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